GSM/D-21 **924**

STATICS

Paper–BM-233

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Two forces of magnitudes 8N and 6N act at a point and the angle between them is 60°. Find the magnitude and direction of their resultant. 2

   (b) Write the condition of equilibrium of a system of coplanar forces acting on a rigid body. 1

   (c) Find C.G. of uniform triangular lamina. 2

   (d) Find the equation of the conjugate line of the given line : 2

   $$\frac{x-f}{t} = \frac{y-g}{m} = \frac{z-h}{n}.$$

   (e) Define stable equilibrium. 1

UNIT-I

2. (a) The resultant of two forces P and Q is of magnitude R. If one of the force be reversed in direction, the resultant is S. Show that:

   $R^2 + S^2 = 2(P^2 + Q^2).$

   (b) Three forces P, Q and R acting at a point O are in equilibrium and the angle between P and Q is double between P and R. Show that:

   $R^2 = Q(Q - P).$

3. (a) Two unlike parallel forces P and Q $(P > Q)$, x meter apart act at two points of a rigid body. Show that if direction of P be reversed, the resultant is displaced through a distance $\dfrac{2PQ}{P^2 - Q^2} x$ metres.

(b)   Forces of magnitude 1, 2, 3, 4, $2\sqrt{2}$ act respectively along the sides AB, BC, CD, DA and diagonal AC of the square ABCD of side a. Show that their resultant is a couple and find its moment.

4.   (a)   A uniform rod AB of length 'a' hangs with one end A against a smooth vertical wall, being supported by a string of length 'l' attached to the other end of the rod and to a point C of the wall vertically above A. Show that if the rod rests inclined to the wall at an angle θ, then $cos^2θ = \dfrac{l^2 - a^2}{3a^2}$. Also find the limits for 'l' and 'a' for equilibrium to be possible.

     (b)   Two equal beams AB, AC each of weight W connected by a hinge at A are placed in a vertical plane with their extremities B, C resting on a horizontal plane, they are kept from falling by string connecting B and C with the middle points of opposite beams. Show that the tension of each string is $\dfrac{W}{8} \sqrt{9cot^2θ + 1}$, θ being the inclination to the horizon of each beam.

5.   (a)   Find how high a particle rest inside a hollow sphere of radius 'r' if the coefficient of friction be $\dfrac{1}{\sqrt{3}}$.

     (b)   The distance of the angular points and intersection of the diagonals of a plane quadrilateral lamina from any line ox in its plane are a, b, c, d and e. Show that the distance of the centre of gravity from the same line is $\dfrac{1}{3}(a + b + c + d + e)$.

UNIT-III

6.   (a)   Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If the weight W be attached to C and the system be suspended from A. Show that there is a thrust in BD equal to $\dfrac{W}{\sqrt{3}}$.

     (b)   A uniform beam of length 2a rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall.

Show that in the position of equilibrium, the rod is inclined to the wall at an angle $\sin^{-1}\left(\dfrac{b}{a}\right)^{\frac{1}{3}}$.

7. (a) Find the equation of the central axis of any given system of forces acting on a rigid body.

(b) Two forces P and Q act along the straight lines whose equation are $y = x$, $z = c$ and $y = -x$, $z = -c$ respectively. Show that their central axis lies on the straight line :

$$y = x\,\frac{P-Q}{P+Q} \quad \text{and} \quad \frac{z}{c} = \frac{P^2-Q^2}{P^2+Q^2}\,.$$

UNIT-IV

8. (a) Show that a given system of forces can be replaced by two forces, equivalent to the given system, in an infinite number of ways and the tetrahedron formed by the two forces is of constant volume.

(b) Show that the minimum distance between two forces which are equivalent to a given system (R, K) and which are inclined at a given angle 2α is $\dfrac{2K}{R}\,cot\,\alpha$ and the forces are then each equal to $\left(\dfrac{R}{2}\right)\sec\alpha$.

9. (a) Find the null point of the plane $lx + my + nz = 1$ for the system of forces $(X, Y, Z; L, M, N)$.

(b) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable.