GSM/D-21 923

PARTIAL DIFFERENTIAL EQUATION

Paper–BM-232

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1\. (i) Form partial differential equation by eliminating arbitrary constants from :

$$z = ax^3 + by^3$$ 1½

(ii) Examine the compatibility of equations :

$$xp - yq = x \text{ and } x^2p + q = zx$$ 1½

(iii) Write down Charpit's Auxiliary equations for : 2

$$z^2 = 1 + p^2 + q^2$$

(iv) Solve the equation by direct integration : 1½

$$\frac{\partial^2 z}{\partial x^2} = xy$$

(v) Classify the differential equation : 1½

$$\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$$

UNIT-I

2\. (a) Obtain partial differential equation by eliminating the arbitrary functions from : 4

$$z = f(xy) + g\left(\frac{x}{y}\right).$$

(b) Solve : 4

$$z(p - q) = z^2 + (x + y)^2.$$

3. (a) Find the complete integral of : 4

$p_1x_1 + p_2x_2 = p_3^2$ by using Jacobi's Method.

(b) Solve the equation $2z + p^2 + qy + 2y^2 = 0$. 4

UNIT-II

4. (a) Solve $(D - D' - 1)(D - D' - 2)\, z = \sin(2x + 3y)$. 4

(b) Solve $(D^3 - 7DD'^2 - 6D'^3)\, z = x^2 + xy^2 + y^3 + \cos(x - y)$. 4

5. (a) Solve : 4

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = (x^2 + y^2)^{n/2}$$

(b) Solve : 4

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$$

UNIT-III

6. (a) Classify and reduce the equation

$$\frac{\partial^2 z}{\partial x^2} - 5 \frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} = 0$$ to canonical form. 4

(b) Solve $r = a^2 t$. 4

7. (a) Find the solution of equation $2s + (rt - s^2) = 1$ by Monge's method. 4

(b) Find the solution of equation $t - r \sec^4 y = 2q \tan y$. 4

UNIT-IV

8. (a) Determine the characteristic of the equation : 4

$$e^{2x} \frac{\partial^2 u}{\partial x^2} + 2e^{x+y} \frac{\partial^2 u}{\partial x \partial y} + e^{2y} \frac{\partial^2 u}{\partial y^2} = 0$$

(b) Solve :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

which satisfy the boundary conditions u(o, y) = u(a, y) = u(x, o) = 0

and u(x, b) = $\sin \dfrac{n\pi x}{a}$. 4

9. (a) Find the solution of 4

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t},$$

$0 < x < 1$, $t > 0$ for which $u(0, t) = u(1, t) = 0$ and $u(x, 0) = k \sin 2\pi x$.

(b) Find the solution of the equation

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$ satisfying the conditions

$u(o, y, t) = u(\pi, y, t) = u(x, o, t) = u(x, \pi, t) = 0$ and $u(x, y, o) = xy(\pi - x)(\pi - y)$. 4