GSM/D-21 **887**

ADVANCED CALCULUS

Paper–BM-231

Time Allowed : 3 Hours] [Maximum Marks : 27

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory.

Compulsory Question

1. (a) State Rolle's Theorem. 1

(b) Find $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial u}{\partial y}$ for functions $u = e^{xy}$. 2

(c) Examine for extreme value $f(x, y) = 3x^2 - y^2 + x^3$ at (0, 0). 2

(d) Find the equation of tangent line at the point $t = 1$ to the curve

$x = 3 + t, \; y = -t^2, \; z = 3 + t^2$. 2

UNIT-I

2. (a) If $f(x) = \begin{cases} \frac{|x-2|}{2-x} & , \; x \neq 2 \\ -1 & , \; x = 2 \end{cases}$, find whether f is continuous at $x = 2$. 2½

(b) Prove that the function $f(x) = 2x^2 + 3x - 4$ is uniformly continuous on [–2, 2]. 2½

3. (a) Examine the applicability of Rolle's theorem for the function

$f(x) = (x^2 - 4x + 3)\, e^{2x}$ in [1, 3]. 2½

(b) $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x}$ 2½

UNIT-II

4. (a) Show that the function $\lim_{(x,y) \to (0,0)} \frac{xy^2}{x^2+y^4}$

(x, y) ≠ (0, 0) is discontinuous at (0, 0). 2½

(b) If $u = sin^{-1}\left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right]^{1/2}$, prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u).$$ 2½

5. (a) $Z(x + y) = x^2 + y^2$, show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$. 2½

(b) Expand $e^x \cos y$ in terms of x and y as far as terms of third degree. 2½

UNIT-III

6. (a) If $f : R^2 \to$ R be a function such that both fx and fy are differentiable at a point (a, b) of the domain, then prove that $fxy(a, b) = fyx(a, b)$. 2½

(b) Show that function $f(x, y) = \sqrt{x^2 + y^2}$ is continuous at point (0, 0) but fx, fy do not exit at (0, 0). 2½

7. (a) Find the values of x, y, z for which

$$\frac{5xyz}{x + 2y + 4z}$$

is maximum given that $xyz = 8$. 2½

(b) Find the extreme values of the function $x^2 + y^2 + z^2$ subject to the condition $xy + yx + zx = 3a^2$. 2½

UNIT-IV

8. (a) Find the osculating plane at (x_1, y_1, z_1) curve of intersection cylinders $x^2 + z^2 = a^2$, $y^2 + z^2 = b^2$. 2½

(b) Find the curvature and torsion of the curves $x = a$ cos$t$, $y = a$ sin$t$, $z = at$ cotα. 2½

9. (a) Show that $\tau = \frac{[\vec{r}' \ \vec{r}'' \ \vec{r}''']}{|\vec{r}' \ \vec{r}''|^2}$ 2½

(b) Find the unit normal vector to the surface $2xz^2 - 3xy - 4x = 7$, at the point (1, –1, 2). 2½