Roll No. ..... Total Pages : 04

GSE/D-21 784

SOLID GEOMETRY

BM-113

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Find the nature of the conic :

$$2x^2 - 72xy + 23y^2 - 4x - 28y - 48 = 0.$$ **2**

(b) Find the equation to the sphere through the circle $x^2 + y^2 + z^2 = a^2$, $z = 0$ and the point (α, β, γ). **2**

(c) Find the equation of the cone circumscribing the sphere $x^2 + y^2 + z^2 - 2x + 2y - 2 = 0$ with its vertex at (1, 1, 1). **2**

(d) Find the equation of the plane which cuts the paraboloid $2x^2 - y^2 = 2z$ in a conic with its centre at the point (2, 3, 4). **2**

Section I

2. Trace the conic :

$$9x^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0.$$ **8**

3. (a) Find the equation of the parabola which touches the conic $x^2 + xy + y^2 - 2x - 2y + 1 = 0$ at the point where it is cut by the line $x + y + 1 = 0$. **4**

(b) Find the conics confocal with $x^2 + 2y^2 = 2$ which passes through the point (1, 1). **4**

Section II

4. (a) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point P (1, – 2, 1) and cuts orthogonally the sphere :

$$x^2 + y^2 + z^2 - 4x + 6y + 4 = 0.$$ **4**

(b) Find the equation to the right circular cone, whose vertex is P (2, – 3, 5), axis PQ which makes equal angles with the axes and which passes through A (1, – 2, 3). **4**

5. (a) Find the condition that the plane $lx + my + nz = 0$ may touch the cone $4x^2 - y^2 + 3z^2 = 0$. **4**

(b) Find the equation of a right circular cylinder of radius 2, whose axis passes through (1, 2, 3) and has direction cosines proportional to 2, 1, 2. **4**

Section III

6. (a) Find the equations of the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the lines $x + 9y - 3z = 0$, $3x - 3y + 6z - 5 = 0$. **4**

(b) Prove that the sum of the squares of the reciprocals of any *three* mutually perpendicular diameters of an ellipsoid is constant. **4**

7. (a) Prove that the six normals from a point to an ellipsoid lie on a curve of second degree. **4**

(b) Show that the plane $x + 2y + 3z = 2$ touches the conicoid $x^2 - 2y^2 + 3z^2 = 2$ and find the point of contact. **4**

Section IV

8. (a) Find the conditions that any *two* lines $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$; $\frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$ be the axes of the section of the conicoid $ax^2 + by^2 + cz^2 = 1$, by a plane through them. **4**

(b) Find the equations to the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, which pass through the point $(a\cos\alpha, b\sin\alpha, 0)$. **4**

9. Reduce to standard form :

$$2y^2-2yz+2zx-2xy-x-2y+3z-2=0$$

and state the nature of surface represented by the equation. **8**