Roll No. .... **Total Pages : 04**

GSE/D-21 **782**

ALGEBRA

BM-111

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) The diagonal elements of a skew-symmetric matrix are all zero. **1½**

(b) A set which contains the null vector '0' is linearly dependent. **1½**

(c) Prove that 0 is a eigen root of a matrix if and only if A is singular. **1½**

(d) Find an equation whose roots are four times the roots of the equation $x^3+2x^2+3x-5=0$. **2**

(e) Show that the equation $x^8+5x^3+2x-3=0$ has at least *six* imaginary roots. **1½**

Section I

2. (a) Prove that every Hermitian matrix A can be written as A = B + *i*C, where B is real and symmetric and C is real and skew-symmetric. **4**

(b) Express $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ as the product of elementary matrices. **4**

3. (a) Find the characteristic vectors of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}.$$ **4**

(b) Any two characteristic vectors corresponding to two distinct characteristic roots of a Hermitian matrix are orthogonal. Prove. **4**

Section II

4. (a) For what value of λ, the equations :

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

have a solution and solve them completely in each case. **4**

(b) If A is a real skew-symmetric matrix such that $A^2 + I = O$, show that A is orthogonal and is of even order. **4**

5. (a) Reduce the bilinear form :

$$x_1y_1 + x_1y_3 - x_2y_1 + x_2y_2 + x_3y_3$$

to the canonical form. Also find the equations of transformations. **4**

(b) Determine the definiteness of the following quadratic form in $\mathbb{R}^3$ with the help of leading principal minors :

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1.$$ **4**

Section III

6. (a) If the product of two roots of the equation :

$$x^4 + px^3 + qx^2 + rx + s = 0,$$

be equal in magnitude but opposite in sign to the product of the other two, show that :

$$p^2s + r^2 = 4qs.$$ **4**

(b) Solve the equation $15x^4 - 8x^3 - 14x^2 + 8x - 1 = 0$, given that the roots are in H.P. **4**

7. (a) Solve the equation $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$, given that the difference between two roots is unity. **4**

(b) Find the equation of squared differences of the roots of the equation $x^3 - 7x + 6 = 0$. **4**

Section IV

8. (a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. **4**

(b) Solve the equation $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method. **4**

9. (a) Solve by the method of resolution into quadratic factors $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$. **4**

(b) Show that the equation $x^3 + x^2 - 2x - 1 = 0$ has three real roots. **4**