Roll No. .... **Total Pages : 04**

GSE/D-21 **745**

ALGEBRA

BM-111

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) If A and B are invertible matrices of order 3, then prove that $(AB)^{-1} = B^{-1}A^{-1}$. **1½**

(b) Prove that determinant of a unitary matrix has absolute value one. **1½**

(c) Prove that eigen values of A^2 are squares of the eigen values of A. **1½**

(d) Form an equation with rational coefficients two of whose roots are $1+5i$ and $1-5i$. **1½**

(e) State Descartes' rule of sign. **1**

Section I

2. (a) Prove that every square matrix A can be expressed in one and only one way as P + *i*Q, where P and Q are Hermitian metrices. **2½**

(b) Using elementary operations, Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 5 & 2 & 3 \end{bmatrix}$. **2½**

3. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. **2½**

(b) Prove that the characteristic roots of a skew Hermitian matrix are either zero or purely imaginary. **2½**

Section II

4. (a) For what values of λ, μ the system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

has no solution, a unique solution and an infinite number of solutions. **2½**

(b) If A is skew Hermitian and (A – I) is non-singular, show that $P = (A + I)(A - I)^{-1}$ is unitary. **2½**

5. (a) Diagonalize :

$$2x_1^2 + 2x_2^2 + 3x_3^2 - 4x_2x_3 + 2x_1x_2 - 4x_3x_1$$

by Lagrange's method. Also find the equations of transformation. **2½**

(b) Prove that the following quadratic form in R^3 is positive definite :

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1.$$ **2½**

Section III

6. (a) Solve the equation $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$ given that the difference of two of the roots is equal to the difference of the other two. **2½**

(b) Solve the equation $3x^3 - 19x^2 + 33x - 9 = 0$ which has repeated roots. **2½**

7. (a) Reduce the cubic equation :

$$2x^3 - 3x^2 + 6x - 1 = 0$$

to the form $z^3 + 3Hz + G = 0$, where H, G are integers. **2½**

(b) If α, β, γ are the roots of cubic equation $x^3 + ax^2 + bx + c = 0$, find the equation whose roots are $\frac{\alpha}{\beta+\gamma}, \frac{\beta}{\gamma+\alpha}, \frac{\gamma}{\alpha+\beta}$. **2½**

Section IV

8. (a) Solve by Cardon's method :

$$x^3 - 3x^2 + 12x + 16 = 0.$$ **2½**

(b) Solve by Descarte's method the equation :

$$x^4 - 3x^2 + 42x - 40 = 0.$$ **2½**

9. (a) Solve by Ferraris' method, the equation :

$$x^4 + 2x^3 - 7x^2 - 8x + 12 = 0.$$ **2½**

(b) Show that for all values of c, the equation $x^5 + 5x^2 + 3x + c = 0$ has at least *two* imaginary roots. **2½**