

Roll No.

Total Pages : 04

MDQ/D-21

5015

ANALYTICAL MECHANICS AND
CALCULUS OF VARIATIONS
MM-502

Time : Three Hours]

[Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section and the compulsory question.

Section I

1. (a) Obtain the Euler's equation for the functional which depends upon one dependent and two independent variables. **8**

- (b) Find the extremal of the functional : **8**

$$I[y, z] = \int_0^1 (y'^2 + z'^2) dx$$

with $y(0) = 0$, $z(0) = 0$, $y(1) = 1$, $z(1) = 2$.

2. (a) Find the natural boundary conditions for the functional $I[y] = \int_a^b f(x, y, y') dx$, which attains extremal curve $y(x)$. **8**

- (b) Find the shortest distance curve from the point (x_1, y_1) to the line $y = mx + b$.

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Section II

3. (a) Define the following terms : **8**
(i) Stationary Constraints
(ii) Holonomic and non-Holonomic constraints
(iii) Scleronomic Constraints
(iv) Rheonomic Constraints
(b) Derive Lagrange's equation of first kind. **8**

4. (a) Prove that :

$$\frac{dE}{dt} = \sum_{i=1}^n \tilde{Q}_i \dot{q}_i + \frac{\partial \pi}{\partial t}, \quad \frac{dE}{dt} = \sum_{i=1}^n \tilde{Q}_i \dot{q}_i$$

and $E = \text{constant}$, for different system of motion. **8**

- (b) Prove that the generalised forces : **8**

$$Q_i = \frac{\partial \pi_i}{\partial t} + \sum_{k=1}^n \left(\frac{\partial \pi_i}{\partial q_k} - \frac{\partial \pi_k}{\partial q_i} \right) \dot{q}_k - \frac{\partial \pi}{\partial q_i}$$

Section III

5. (a) Prove that the necessary and sufficient condition for function $f(t, q_i, p_i)$ to be the integral of equation of motion is : **8**

$$\frac{\partial f}{\partial t} + (f, H) = 0$$

- (b) State and prove Hamilton's principle. **8**

6. Derive Jacobi's equation. 16

Section IV

7. (a) State and prove Jacobi's theorem. 8
 (b) Show that the transformation : 8

$$\tilde{q}_i = \alpha p_i, \tilde{p}_i = \beta q_i$$

$(i = 1, \dots, n), \alpha \neq 0, \beta \neq 0$ is canonical with
 $\tilde{H} = -\alpha\beta H$.

8. Define Lagrange's bracket and show that : 16

$$[q_i, q_k] = 0, [p_i, p_k] = 0, [q_i, p_k] = c\delta_{ik}$$

$(i, k = 1, \dots, n)$, where c is the valence of canonical transformation expressing the necessary and sufficient condition for the transformation to be canonical.

Compulsory Question

9. (i) Define a functional with example. 2
 (ii) Find the extremal of the functional : 2

$$I[y] = \int_0^1 (y'^2 - y^2) dx$$

with $y(0) = 0$ and $y(1) = 1$.

- (iii) Define degree of freedom. 2
 (iv) Prove that : 2

$$(C\phi\psi) = C(\phi\psi),$$

where C is constant.

- (v) Define free canonical transformation. 2
- (vi) What is general equation of dynamics ? 2
- (vii) State principle of virtual work. 2
- (viii) Prove that : 2

$$T = T_2 + T_1 + T$$