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MDE/ D-21: 4318

MM-405: Differential Equations-I

Time: 3 Hours] **[Max. Marks: 80**

	Note: Attempt five questions in all, selecting one question from each section. Question No. I is compulsory.	
I(a)	Explain a 0.0001-approximate solution of an initial value problem (IVP).	2
(b)	Is every uniformly continuous function on an interval I equicontinuous? Justify your answer by citing an example.	2
(c)	Find a fundamental matrix of the linear system $x_1'(t) = 34\log(t)\, x_1(t), x_2'(t) = 4\, t^2\, x_2(t)$.	2
(d)	Find a normalized form of a differential equation whose fundamental set is $\{2x^4, e^{-3x}\}$ over the interval $(1, \infty)$.	2
(e)	If $x(t) \in R^n$ over an interval I, then prove that $\left\lVert \int_a^b x(t)\,dt \right\rVert \le \int_a^b \lVert x(t) \rVert dt$	2
(f)	Verify Lagrange's identity for an example of third order linear differential equation.	2
(g)	Given a differential equation $\frac{d^3y}{dx^3} - 5x\, \log x \frac{dy}{dx} + 6x^2 \sin x\; y = x^3$. Find its equivalent system of ODEs.	2
(h)	Explain maximal and minimal solutions by giving one example of each.	2
	Section – I	
II(a)	State the conditions under which for an initial value problem $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$, there exists a sequence of approximate solutions which is uniformly convergent on some interval [a, b]. Prove the so stated result.	8
(b)	Solve the initial value problem $x'(t) = 6x^3 + 5x - 20, x(0) = c$ by method of successive approximations.	8
III(a)	When will an IVP have a unique solution? Prove your claim.	8
(b)	Prove that every equicontinuous and uniformly bounded sequence of functions defined on a bounded interval I has a subsequence which is uniformly convergent on I.	8
	Section – II	
IV(a)	Let $Y(t)$ be a continuous $n \times n$ matrix function defined on a closed and bounded interval I. Prove that there exists a unique solution of the initial value problem $x'(t) = Y(t)x(t), x(t_0) = x_0; t, t_0 \in I$; on I.	8
(b)	Define adjoint system of a linear system of ODEs and then derive its fundamental matrix. Is there any relation between the fundamental matrices of these two systems?	8

V(a)	Prove that every fundamental matrix of a periodic linear system of ODEs can be written as a product of a periodic non-singular matrix, with same period, and e^{tC}, where C is a constant matrix.	8
(b)	Find the solution of the linear system $\begin{pmatrix} x_1'(t) \\ x_2'(t) \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} e^t \\ e^{-t} \end{pmatrix}$ such that $\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.	8
	Section – III	
VI (a)	If $f_1, f_2, \cdots, f_n$ are n solutions of the equation $L_n\, x(t) = 0$ on [a,b], then prove that their wronskian is either zero everywhere or nowhere on $[a, b]$.	8
(b)	Given n real constants and a homogeneous linear differential equation of order n, then show that there exists a solution whose value and values of its first (n-1) derivatives at some fixed point are the given real constants.	8
VII(a)	Find the general solution of the differential equation $(x^3 - x^2)y''(x) - (x^3 + 2x^2 - 2x)y'(x) + (2x^2 + 2x - 2)y = 0$ by reducing its order.	8
(b)	If $\lambda_1, \lambda_2, \dots, \lambda_s$ are the distinct characteristic roots, with multiplicities $m_1, m_2, \dots, m_s$ respectively, of the differential equation with constant coefficients $x^{(n)} + b_1 x^{(n-1)} + \cdots + b_n x = 0$, then prove that its fundamental set is given by $t^k e^{t\lambda_i}$ $(k = 0,1,2, \dots, m_i - 1; i = 1,2, \dots, s)$.	8
	Section – IV	
VIII	Let $f \in (C, Lip)$ in a domain D of the (n+1)-dimensional (t, x) space, f_x exists and $f_x \in C$ on D and suppose that ψ is solution of $x'(t) = f(t,x)$ on an interval I = [a,b]. Prove that there exists a $\delta > 0$ such that for any $(\tau, \xi) \in U$, where $U:\ a < \tau < b, \lvert\xi - \psi(\tau)\rvert < \delta$ there exists a unique solution φ of the given differential equation on I with $\varphi(\tau, \tau, \xi) = \xi$. Also prove that $\varphi \in C^1$ on the (n+2)- dimensional set $V:\ a < t < b, (\tau, \xi) \in U$ and moreover $\det\varphi_\xi(t, \tau, \xi) = \exp[\int_\tau^t tr f_x(s, \varphi(s, \tau, \xi))ds]$.	16
IX(a)	Prove that any nth order non-linear differential equation can be expressed in terms of a system of differential equations.	8
(b)	State and prove Nagumo's theorem.	8