Roll No. .... **Total Pages : 04**

MDE/D-21 4316

TOPOLOGY–I

MM-403

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **9** is compulsory. All questions carry equal marks.

Section I

1. (a) Show that intersection of two topologies is again a topology. Is it true for union also ? Explain.

(b) Let C^* be a closure operator defined on a set X. Let F be the family of all subsets F of X for which $C^*(F) = F$ and let T be a family of all complements of members of F. Then prove that T is a topology for X and if C is the closure operator defined by the topology T, then $C^*(E) = C(E)$ for all subsets $E \subseteq X$.

2. (a) Prove that a family **B** of sets is a base for some topology for the set $X = U\{B; B \in \mathbf{B}\}$ if and only if for every $B_1, B_2 \in B$ and $x \in B_1 \cap B_2$, there exists a $B \in \mathbf{B}'$ such that $x \in B \subseteq B_1 \cap B_2$.

(b) Let (X, T) and (X^*, T^*) be the two topological spaces and $f : X \to X^*$. Prove that the following conditions are each equivalent to the continuity of f on X :

(i) The inverse image of every open set in X* is an open set in X.

(ii) The inverse image of every closed set in X* is a closed set in X.

(iii) $f(C(E)) \subseteq C^*(f(E))$ for every $E \subseteq X$.

Section II

3. (a) Show that the property of a space being a T_0-space is preserved under one to one, onto, open mapping and hence is a topological property.

(b) Prove that a T_1-space X is countably compact if and only if every countable open covering of X is reducible to a finite subcover.

4. (a) Prove that an infinite Hausdorff space X contains an infinite sequence of non-empty disjoint open sets.

(b) Prove that the property of a space being T_4 is Hereditary.

Section III

5. (a) Prove that complete regularity is Hereditary property.

(b) Prove that a topological space is completely regular if and only if the family of all continuous real valued functions on it distinguishes points from closed sets.

6. (a) Prove that if the open set G has a non-empty intersection with a connected set C in a T_4-space X, then either C consists of the only point or the set $C \cap G$ has cardinality greater than or equal to cardinality of reals.

(b) Prove that a net (x_λ) has the point x as a cluster point if and only if it has a subnet which converges to x.

Section IV

7. (a) If E is a subset of a subspace (X^*, T^*) of a topological space (X, T), then prove that E is T^* compact if and only if it is T-compact.

(b) Prove that a topological space (X, T) is compact if and only if any family of closed sets having the finite intersection property has a non-empty intersection.

8. (a) Prove that $X \times Y$ is compact if and only if X and Y are compact.

(b) Let $P : \Lambda \to X$ be a net and **H** the filter associated with it. Let $x \in X$. Then prove that P converges to x as a net if and only if **H** converges to x as a filter. Also, x is a cluster point of the net P if and only if x is a cluster point of the filter **H**.

Section V

9. (a) Define countable complement topoplogy.

(b) Define closed topological space.

(c) Define weakly hereditary property.

(d) State finite intersection property.

(e) Define first and second countable space.

(f) Explain with example that every separable space need not be second axiom.

(g) Define Ultrafilters.