

Time : Three Hours]

[Maximum Marks : 80

**Note :** Attempt *five* questions in all, selecting at least *one* question from each section. Question No. 1 is compulsory. All questions carry equal marks.

**Compulsory Question**

1. (a) Find power set of  $\{2,3,4\}$ .

(b) Define Lattices.

(c) Evaluate :  $\lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$ .

(d) Find order and degree of the differential equation

$$\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^{1/3} = xy.$$

(e) Solve the differential equation:

$$\frac{d^3y}{dx^3} - 3 \frac{d^2y}{dx^2} + 3 \frac{dy}{dx} - y = 0.$$

**SECTION-I**

2. (a) Prove that,  $A - (B \cap C) = (A - B) \cup (A - C)$ .

- (b) Let  $A = \{1, 2, 3, 4\}$  and  
 $R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$ . Show that  $R$  is an equivalence relation.

3. (a) In how many ways 5 different microprocessor books and 4 different digital electronics books be arranged in a shelf so that all the four digital electronics books are together.
- (b) If  $f(x, y, z) = (xvy) \wedge (xvy') \wedge (x'vz)$  be a given Boolean function. Determine its DN form.

### SECTION-II

4. (a) By using  $\varepsilon - \delta$  definition of limit show that

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2, x \neq 1.$$

- (b) Find the value of  $a$  if the function  $f$  given by

$$f(x) = \begin{cases} 2x - 1, & 2 < x \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$$

is continuous at  $x = 2$ .

5. (a) Find  $\frac{dy}{dx}$  if  $y = \frac{\log x}{1 + \log x}$ .

- (b) If  $y = x^x$ , show that  $\frac{d^2y}{dx^2} - \frac{1}{y} \left( \frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$ .

### SECTION-III

6. (a) Form the differential equation of the equation  $y = ax^3 + bx^2$  by eliminating the arbitrary constants  $a$  and  $b$ .

- (b) Solve the differential equation :

$$\frac{dy}{dx} = (4x + y + 1)^2, \text{ if } y(0) = 1.$$

7. (a) Solve the differential equation :

$$x - xdy = \sqrt{x^2 + y^2} dx.$$

- (b) Solve the differential equation :

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$$

### SECTION-IV

8. (a) Solve the differential equation :

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + \sin 2x.$$

- (b) Solve the differential equation :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x.$$

9. (a) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = \log x.$$

- (b) Determine the curve whose sub-tangent is twice the abscissa of the point of contact and passes through the point (1, 2).