Roll No. .... Total Pages : 3

GSQ/D-21 **1063**

MATHEMATICS
(Groups and Rings)
Paper–BM-352

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Prove that a group G is abelian if every element of G except the ideatity element is of order 2. 1½

(b) Let G be a cyclic group of order 4. Show that group of automorphisms of G is of order 2. 1½

(c) Show by an example that unity of a ring and its subring is not same. 1½

(d) Let R be an Euclidean ring and *a, b* be two non-zero elements of R. Then $d(ab) = d(a)$ if b is a unit in R. 1½

(e) Show that the ideal $S = \{6r : r \in z\}$ is not a prime ideal of the ring of integers. 2

SECTION–I

2. (a) If H and K are two subgroups of a group G then show that HK is a sub group of G iff HK = KH. 4

(b) If an abelian group of order 6 contains an element of order 3, show that it must be a cyclic group. 4

3. (a) If M and N are normal subgroups of a group G, prove that MN is also of normal subgroup of G. 4

(b) Show that the orders of the elements a and $x^{-1}ax$ are the same, where a, x are the two elements of a group. 4

SECTION–II

4. (a) Show that an infinite cyclic group G is isomorphic to the additive group of integers. 4

(b) Show that every inner automorphism of a group is automorphism of that group. 4

5. (a) Show that centre of a non-abelian group of order 343 always have 7 elements in its centre. 4

(b) Write all permutations of $S = \{1, 2, 3, 4\}$ list even and odd permutations. 4

SECTION–III

6. (a) Show that every finite non-zero integral domain is a field. 4

(b) If S_1 and S_2 be two ideals of a ring R, then $S_1 + S_2$ is the smallest ideal containing $S_1 \cup S_2$. 4

7. (a) Show that S is an ideal of S + T where S is any ideal of a ring R and T is any subring of R. 4

(b) Let $S \subseteq T$ be two ideals of a ring R, then $R/T \cong \dfrac{R/S}{T/S}$. 4

SECTION–IV

8. (a) Let R be an euclidean ring. Show that any two elements a and b in R have a greatest common divisor. 4

(b) Show that every non-zero prime ideal of a principal ideal domain is maximal. 4

9. (a) Show that if 'a' is an irreducible element of a unique factorization domain. R, then 'a' must be prime. 4

(b) Show that the polynomial $x^4 + 1$ is irreducible over Q. 4