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GSQ/D-21 **1037**

MATHEMATICS
(Groups and Rings)
Paper–BM-352

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all, selecting *one* question from each section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Prove that a group G is abelian if every element of G except the identity element is of order 2. 1

(b) Prove that an ideal S of a ring R is a subring of R. 1

(c) Define primitive polynomial and irreducible polynomial with example. 1

(d) How many generators are there in a cyclic group of order 10 ? 1

(e) If $f : G \to G'$ is homomorphism of group then show that $f(e) = e'$ where e and e' are identity of G and G′ respectively. 2

UNIT–I

2. (a) If G′ is a commutator subgroup of G then prove that G/G′ is abelian. 2½

(b) If H is the only subgroup of finite order in the group G, then prove that H is the normal subgroup of G. 2½

3. (a) Prove that every finite group of prime order is cyclic. 2½

(b) Show that the set $G = \left\{a + b\sqrt{2} : a, b \in Q\right\}$ is an abelian group with respect to addition. 2½

UNIT–II

4. (a) If G is a finite abelian group of order n and m is a positive integer such that $(m, n) = 1$, then show that $f : G \to G$ defined by $f(x) = x^n$ is an automorphism. 2½

(b) Show that if G is a non-abelian group of order 343 then O (Z(G)) = 7. 2½

5. (a) Let Z(G) be the centre of a group G then $a \in Z(G)$ iff $N(a) = G$. 2½

(b) Let $f = (1\ 2\ 3)$, $g = (4\ 5)$ be two cyclic permutations defined on set S = {1, 2, 3, 4, 5} prove that

$f \cdot g = g \cdot f$. 2½

UNIT–III

6. (a) Prove that characteristic of an integral domain is either zero or prime number. 2½

(b) Prove that intersection of any two left ideal of a ring is a left ideal of the ring. 2½

7. (a) Prove that if I and J are two ideal of a ring R. Then $(I + J)/ J \cong J/I \cap J$. 2½

(b) Let R be a commutative ring with unity. If R has no proper ideal then R is a field. 2½

UNIT–IV

8. (a) Let R be an integral domain with unity element. If a, b are two non-zero element of R then $a \sim b$ iff a/b and b/a. 2½

(b) Show that $\sqrt{-5}$ in a prime element of the ring

$Z\sqrt{-5} = \{a + \sqrt{-5}\, b : a, b \in Z\}$. 2½

9. (a) If F is a field, then F[X] may not be a field. 2½

(b) Let R be an integral domain with unity. Then units of R and R[X] are some. 2½