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GSQ/D-21 **1036**

MATHEMATICS
(Real Analysis)
Paper–BM-351

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, select *one* question from each Section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Give an example, without proof, of a function which is Riemann integrable on [0, 1] but is not monotonic on [0, 1]. 1

(b) Evaluate the improper integral $\int_0^1 \frac{dx}{x^2}$. 1

(c) Define semi-metric space. 1

(d) Define norm of a partition P and give an example to explain it. 1

(e) Find derived set of $A = \left\{\frac{1}{n} : n \in N\right\}$, when (R, d) be the usual metric space. 1

SECTION–I

2. (a) Prove that every continuous function defined on closed interval [a, b] is integrable on [a, b]. 3

(b) Show that $f(x) = 2x + 3$ is integrable on [2, 3] and $\int_2^3 f(x)\,dx = 8.$ 2½

3. (a) Show that $\frac{1}{\pi} \le \int_0^1 \frac{\sin \pi x}{1+x^2}\,dx \le \frac{2}{\pi}.$ 3

(b) If f is integrable on $[a, b]$ and f is the primitive of f on $[a, b]$, then prove that $\int_a^b f dx = F(b) - F(a).$ 2½

SECTION–II

4. (a) Show that the integral $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^m}{x^n}\,dx$ exists iff $n <$ mH. 3

(b) Examine the convergence of $\int_0^{\infty} \frac{x \tan^{-1} x}{(1+x^4)^{1/3}}\,dx.$ 2½

5. (a) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}\,dx,\ a > 0.$ 3

(b) Evaluate $\int_0^{\pi} \frac{\log(1+a\cos x)}{\cos x}\,dx$ for $|a| < 1.$ 2½

SECTION–III

6. (a) Prove that in a metric space, the union of arbitrary collection of open sets is open. 3

(b) Let (x, d) be a metric space. Then prove that d^* defined by $d^*(x, y) = \dfrac{d(x,y)}{1+d(x,y)}$; $x, y, \in x$ is also a metric on x. 2½

7. State and prove Contor's intersection theorem. 5½

SECTION–IV

8. (a) If (X, d) and (Y, d^*) be two metric spaces and f, g be two continuous functions of X into Y then show that the set $\{x \in X : f(x) = g(x)\}$ is closed subset of X. 3

(b) Prove that the usual metric space is not compact. 2½

9. (a) A closed subset of a complete metric space is compact iff it is totally bounded. 3

(b) Prove that closure of a connected subset in (X, d) is also connected. 2½