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GSM/D-21 922

ADVANCED CALCULUS

Paper–BM-231

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) State Implicit Function Theorem. 2

(b) Define Continuous Function and Uniformly Continuous Function. 2

(c) Define Principal Normal and Binormal. 2

(d) Define Involute and Evolute of curves. 2

UNIT-I

2. (a) Every function defined and continuous on a closed interval is bounded in that interval. Prove it. 4

(b) Verify Rolle's theorem for the function : 4

$f(x) = (x^2 - 4x + 3)e^{2x}$ in [1, 3].

3. (a) Evaluate : $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$ 4

(b) Prove that the function defined by: 4

$f(x) = \sin \frac{1}{x}, \; x \in R^+$

is continuous but not uniformly continuous on R^+.

UNIT-II

4. (a) Prove that the function f defined by :

$$f(x, y) = \begin{cases} y \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is continuous at the origin. 4

(b) If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ 4

Prove that

$$\frac{x^2\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 y}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4\, cos^3 u}$$

5. (a) Let f = R² → R be defined as : 4

$$f(x, y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

Prove that $\lim\limits_{(x, y) \to (0, 0)} f(x, y)$ does note exits.

(b) If y³ – 3ax² + x³ = 0, prove $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$ 4

UNIT-III

6. (a) Show by an example that a function of two variables is continuous and possesses firsts order partial derivatives at a point but not differentiable at that point. 4

(b) Examine the function: 4

$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$

for maxima and minima.

7. (a) Find the maximum value of cos A cos B cos C, where A, B, C are the angles of plane triangle ABC. 4

(b) For the function: 4

$$f(x, y) = \begin{cases} \frac{1}{4}(x^2+y^2) \log (x^2+y^2) & ; (x, y) \neq (0, 0) \\ 0 & ; (x, y) = (0, 0) \end{cases}$$

Show that $f_{xy} = f_{yx} \; \forall \; x, y$ but the conditions of Schwarz's theorem are not satisfied.

UNIT-IV

8. (a) Find the length of the circular helix

$\vec{r}(t) = a\ cost\ \hat{i} + a\ sint\ \hat{j} + ct\ \hat{k}\ , -\infty < t < \infty$

from $(a, 0, 0)$ to $(a, 0, 2\pi c)$. Also obtain its equation in terms of parameter 's'. 4

(b) Find the equation of osculating sphere at (1, 2, 3) on the curve

$x = 2t + 1, y = 3t^2 + 2, z = 4t^3 + 3.$ 4

9. (a) Find the curvature and torsion of the helix 4

$x = a \cos t,\ y = a \sin t,\ z = at \tan \alpha.$

(b) Find the envelope of the sphere. 4

$(x - a \cos\theta)^2 + (y - a \sin\theta)^2 + z^2 = b^2.$