

BCA/M21

1889

## MATHEMATICAL FOUNDATIONS-II

Paper-BCA-123

Time allowed : 3 Hours

Maximum Marks : 80

**Note :** Attempt **five** questions in all, selecting **one** question from each unit.  
Question No. **9** is compulsory. All questions carry equal marks.

## UNIT-I

1. (i) Show that :  $[(p \rightarrow q) (q \rightarrow r)] \rightarrow (p \rightarrow r)$  is a tautology. 8
- (ii) Construct the truth table of the following statement : 8
  - (a)  $(p \Leftrightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$       (ii)  $(p \wedge q) \vee \sim(p \vee q)$ .
2. (i) Prove by the principle of Mathematical Induction that the sum of first  $n$  natural number is  $\frac{n(n+1)}{2}$  . for all  $n \in N$ . 8
- (ii) For all  $n \in N$ , show that  $11^{n+2} + 12^{2n+1}$  is divisible by 133. 8

## UNIT-II

3. (i) Show that the set  $G = \{-1, 1, -i, i\}$  is a group with respect to multiplication. 8
- (ii) Let  $G = \{0, 1, 2, 3, 4\}$ . Find the order of the elements of the groups  $G$  under the binary operation 'addition modulo 5.' 8
4. (i) Prove that the set of Rational numbers is a field with respect to addition and multiplication. 8
- (ii) Prove that the necessary and sufficient conditions for a non-empty subset  $S$  of ring  $R$  to be a subring of  $R$  are
  - (a)  $a, b \in S \Rightarrow a - b \in S$       (b)  $a, b \in S \Rightarrow a \cdot b \in S$ . 8

## UNIT-III

5. (i) Find the inverse of the matrix :  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$  8
- (ii) Find  $X$  and  $Y$  if  $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$  and  $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ . 8

6. (i) Find the rank of the following matrix : 8
- $$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$$
- (ii) Using matrix method, solve the following system of equation : 8
- $$\begin{aligned} x + y + z &= 6 \\ x - y + x &= 2 \\ 2x + y - z &= 1. \end{aligned}$$

#### UNIT-IV

7. Find the characteristics roots and the corresponding vectors for the following matrix : 16
- $$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$
8. Verify Cayley Hamilton theorem and find  $A^{-1}$  for the matrix : 16
- $$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

#### Compulsory Question

9. (i) Identify the quantifiers and write the negation of the statements “There exists a capital for every state in India.” 2
- (ii) Define Normal Subgroups. 2
- (iii) Define Ring with unity. 2
- (iv) Define Skew-symmetric matrix with example. 2
- (v) Find the Spectrum of the matrix : 2
- $$\begin{bmatrix} 2 & 7 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$
- (vi) Prove that ‘O’ is a latent root of a matrix A if A is singular. 2
- (vii) If A is a square matrix then show that  $A + A^{\theta}$  is Hermitian. 2
- (viii) Write composition table for S with respect to multiplication modulo  $\sigma$  where  $S = \{0, 1, 2, 3, 4, 5\}$ . 2