Roll No. .... **Total Pages : 2**

GSQ/M21 **1744**

LINEAR ALGEBRA

Paper–BM-362

Time allowed : 3 Hours **Maximum Marks : 40**

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question No. **1** is compulsory.

Compulsory Question

1. (i) Find the dimension of Vector space $Q(\sqrt{2})$ over Q. 2
   (ii) Define Linear dependence and Independence of vectors of a set. 2
   (iii) Express (1, 2) as a Linear combination of (2, 0) and (1, 3). 1
   (iv) Find the norm of vector u = (2, –3, 6) and normalize the vector. 2
   (v) Define Inner Product Space. 1

UNIT-I

2. (i) Show that the set $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$ is a vector space over Q with respect to the compositions :
   $(a + b\sqrt{2}) + (c + d\sqrt{2}) = a + c + (b + d)\sqrt{2}$
   $\alpha(a + b\sqrt{2}) = a\alpha + b\alpha\sqrt{2}$
   where a, b, c, d and α are rational numbers. 4
   (ii) Union of two subspacer is a subspace if and only if one is contained in the other. 4
3. (i) The intersection of two subspaces w_1 and w_2 of Vector Space $V(F)$ is also a subspace of $V(F)$. 4
   (ii) Determine a basis of the subspace spanned by the vector (–3, 1, 2), (0, 1, 3), (2, 1, 0), (1, 1, 1,). 4

UNIT-II

4. (i) Let $u_1 = (1, 1)$, $u_2 = (0, 1)$ be a basis of IR^2. Let $T : IR^2 \to IR$ to be linear transformation for which $T(u_1) = 3$ and $T(u_2) = -2$. Find the linear transformation T. 4
   (ii) Let $T : U(F) \to V(F)$ be a linear transformation. If $u_1, u_2, ..., u_n$ are linearly independent vectors of U and T is one-one then $T(u_1), T(u_2), ..., T(u_n)$ are also linearly independent. 4

5. (i) If $T : U(F) \rightarrow V(F)$ is a linear transformation, then :
$dim[R(T) + dim\,[N(T)] = dim\,U$. 5

(ii) If $T : U \rightarrow V$ be a homomorphism, then $ker(T)$ is a subspace of U. 3

UNIT-III

6. (i) Let $T: IR^3 \rightarrow IR^3$ be a linear operator defined by :
$T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is invertible and find T^{-1}. 4

(ii) If linear transformation $T : \not\subset(IR) \rightarrow \not\subset(IR)$ defined as $T(a+ib) = a-ib$ for all $a, b \in IR$. Find matrix of T with respect to the ordered basis $B = \{1 + i, 1 + 2i\}$. 4

7. (i) Prove that similar matrices have same characteristic polynomial.

(ii) Find the Eigen values, Eigen vectors for the matrix : 4

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

UNIT-IV

8. (i) State and prove Cauchy Schwarz inequality. 4

(ii) Every finite dimensional vector space is an inner product space. 4

9. (i) Obtain an orthonormal basis with respect to standard inner product for the subspace of IR^3 generated by (1, 0, 1), (1, 0, –1) and (0, 3, 4). 4

(ii) Let T be a Linear Operator on a Unitary space V, then T is normal iff :
$\| T^*(u) \| = \| T(u) \| \;\forall\; u \in V$. 4