Roll No. .... **Total Pages : 3**

GSQ/M21 **1722**

LINEAR ALGEBRA

Paper–BM-362

Time allowed : 3 Hours **Maximum Marks : 26**

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question No. **1** is compulsory.

Compulsory Question

1. (i) In a vector space $V(F)$, prove that : $(-1)u = -u$ for all $u \in V$. 1

(ii) Prove that the set {(1, 0, 0), (0, 1, 0), (0, 0, 1)} is a basis of vector space $R^3(R)$. 1

(iii) Prove that the transformation $T : R^2 \to R$ defined by $T(x, y) = xy$ is not linear. 1

(iv) Define Dual Space. 1

(v) Define Inner Product Space. 1

(vi) Define Self Adjoint Operator. 1

UNIT-I

2. (i) Prove that the necessary and sufficient condition for a vector space V(F) to be a direct sum of its subspaces W1 and W2 are that :

(a) $V = W_1 + W_2$ (b) $W_1 \cap W_2 = \{0\}$. 2½

(ii) Prove that the four vectors $v_1 = (1, 0, -1)$, $v_2 = (-1, 0, 0)$, $v_3 = (1, 0, 1)$ and $v_4 = (2, 1, 3)$ are linearly depended over R. 2½

3. (i) Prove that every subspace W of a finite dimensional vector space $V(F)$, has a complementary subspace W' and $lim\ W' = lim\ V - lim\ W$. 2½

(ii) If V is a vector space of all square matrices over R and $W = \left\{ \begin{bmatrix} a & b \\ o & c \end{bmatrix} : a, b, c, \in R \right\}$. Find a basis of $\frac{V}{W}$. 2½

UNIT-II

4. (i) Prove that every n-dimensional vector space $U(F)$ is isomorphic to F^n. 2½

(ii) If $T : U(F) \rightarrow V(F)$ is a linear transformation, then prove that : Rank T + Nullity $T = lim\ U$. 2½

5. (i) Let $S = \{v_1, v_2, v_3\}$ be a basis of $v_3(R)$, defined by $v_1 = (-1, 1, 1)$, $v_2 = (1, -1, 1)$, $v_3 = (1, 1, -1)$. Find the dual basis of S. 2½

(ii) If V is a finite dimensional vector space and W be a subspace of V, then prove that $A[A(w)] = W$. 2½

UNIT-III

6. (i) Let $T_1 : R^3 \rightarrow R^2$ such that $T_1(x, y, z) = (x + y + z, x + y)$

$T_2 : R^3 \rightarrow R^2$ such that $T_2(x, y, z) = (2x + z, x + y)$

$T_3 : R^3 \rightarrow R^2$ such that $T_3(x, y, z) = (2y, x)$

Find a formula defining the transformation $2T_1 - 3T_2 + 4T_3$. Also find the image of $(-1, 0, 3)$ under this map and show that T_1, T_2, T_3 are linearly independent. 2½

(ii) Let $T : R^3 \rightarrow R^3$ be a linear operator defined by $T(x, y, z) = (x - 3y - 2z, y - 4z, z)$. Show that T is invertible and find T^{-1}. 2½

7. (i) Write the matrix of linear transformation $T : p_3(x) \rightarrow p_2(x)$ defined by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + (a_1 + a_3)x + (a_0 + a_1)x^2$ relative to the basis $B = \{1, x - 1, (x - 1)^2, (x - 1)^3$ and $B^1 = \{1, x, x^2\}$. 2½

(ii) Let $T : R^3 \rightarrow R^3$ be a linear transformation such that :

$$A = \begin{bmatrix} 3 & 1 & 7 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

is a matrix of T with respect to ordered basis {(1, 2, 3), (1, 2, 0), (1, 0, 0)}. Determine the Eigen values and Eigen vectors for T. 2½

UNIT-IV

8. (i) Let V be an inner product space, then prove that : $\| u + v \| \leq \| u \| + \| v \|$. 2½

(ii) Let S be a subset of an inner product space V then show that : $S^1 = S^{111}$. 2½

9. (i) Let W be a subspace of an inner product space $V(F)$. If $\{u_1, u_2, ..., u_n\}$ is an orthonormal basis of W and $\{v_1, v_2, ... v_m)$ is an orthonormal

basis of $W_{\perp}$, then show that $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_m\}$ is an orthonormal basis of V. 2½

(ii) Let T be a normal operator on an inner product space V. If $u \in V$, then show that :

$T(u) = 0$ iff $T^*(u) = 0$. 2½