GSQ/M-21 1721

REAL AND COMPLEX ANALYSIS

Paper–BM-361

Time Allowed : 3 Hours] [Maximum Marks : 27

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory.

Compulsory Question

1. Write short answer of the following :

(a) Prove Symmetry of Beta function. 2

(b) Find the Fourier coefficient for the function f(x) = x in $\left[= \pi, \pi\right]$. 1

(c) Find a point on the complex plane corresponding to the point $\left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$ on the Riemann sphere $x^2 + y^2 + z^2 = 1$. 2

(d) Find the angle of rotation at z = 2 + i for the transformation $w = z^2$. 1

(e) Find the fixed points of Bilinear transformation $w = \frac{z}{z-2}$. 1

UNIT–I

2. (a) Find the Jacobian of u, v, w with respect to x, y, z given that $u = x + y + z$; $v^2 = yz + zx + xy$; $w^3 = xyz$. 2½

(b) Prove that :

$$\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}.$$ 2½

3. (a) Evaluate $\iiint xyz\, dx\, dy\, dz$ over the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$$ 2½

(b) Evaluate $\int_0^{\infty}\int_0^{x} x\, e^{-x^2/y}\, dy\, dx$ by changing the order of integration. 2½

UNIT–II

4. (a) Find the Fourier series for the function

$$f(x)=|\sin x|; -\pi<x<\pi.$$ 2½

(b) Find the half-range cosine series for $f(x)=x(\pi-x)$ in the interval $(0,\ \pi)$. 2½

5. (a) Obtain Fourier series for the function $f(x)=x-x^2,\ -1<x<1$. 2½

(b) Let

$$f(x)=\begin{cases}-1 & , \quad -\pi<x<0 \\ 1 & , \quad 0<x<\pi\end{cases}.$$

Using Parseval's identity, compute the sum $\sum_{k=1}^{\infty}(2k-1)^{-2}$. 2½

UNIT–III

6. (a) Prove that $f(z)=\overline{z}$ is nowhere differentiable, but continuous everywhere in complex plane. 2½

(b) Show that $u=\frac{1}{2}\log\left(x^2+y^2\right)$ is harmonic and find its harmonic conjugate. 2½

7. (a) Prove that an analytic function with constant modulus is constant. 2½

(b) For what value of λ, the function $f(z)=r^2\cos\lambda\theta+ir^2\sin 2\theta$ is analytic. Also find $f'(0)$. 2½

UNIT–IV

8. (a) Let the rectangular region D in the z-plane be bounded by x = 0, y = 0, x = 2, y = 3. Determine the region D^1 of the w-plane into which D is mapped under the transformation $w = \sqrt{2}\, e^{i\frac{\pi}{4}} z$. 2½

(b) Find the image of $|z + 3i| = 6$ under the transformation

$$f(z) = \frac{1}{z}.$$ 2½

9. (a) Find the Bilinear transformation which maps the points z = 0, –1, i onto w = i, 0, ∞. Also find the image of the unit circle $|z| = 1$. 2½

(b) Find all the Mobius transformation which map the half-plane $I(z) \geq 0$ into circle $|w| \leq 1$. 2½