Roll No. ..... Total Pages : 04

GSM/J-21 **1613**

MATHEMATICS

Sequences and Series

BM-241

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Define limit point of a set and give an example of a set which has two limit points. **2**

(b) Define convergence of a sequence and give an example of a convergent sequence. **2**

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \cos \frac{1}{n}$. **2**

(d) Test the absolute convergence of the infinite series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$. **2**

Section I

2. (a) Define closed set. Prove that arbitrary intersection of closed sets is a closed set. **4**

(b) State and prove Archimedean property of reals. **4**

3. (a) Define interior of a set. Prove that interior of a set is an open set. **4**

(b) Show that intersection of any family of compact sets is compact. **4**

Section II

4. (a) Prove that every bounded sequence has a cluster point. **4**

(b) Discuss the convergence of the sequence $< a_n >$ where $a_n = 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \ldots + \frac{1}{2n-1}$. **4**

5. (a) Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$, $x > 0$. **4**

(b) Discuss the convergence of the series :

$$\frac{1}{2} + \frac{\sqrt{2}}{5} + \frac{\sqrt{3}}{10} \ldots + \frac{\sqrt{n}}{n^2+1} + \ldots \ .$$ **4**

Section III

6. (a) State and prove Gauss test for the convergence of an infinite series. **4**

(b) Test the convergence of infinite series :

$(\log 2)^k x^2 + (\log 3)^k x^3 + (\log 4)^k x^4 + \ldots.,\ x > 0$. **4**

7. (a) State and prove Cauchy Integral test for the convergence of an infinite series. **4**

(b) Test the convergence of $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n x^n,\ x > 0$.

4

Section IV

8. (a) Discuss the absolute convergence of the series

$\sum_{n=1}^{\infty} \frac{x^n}{n!}$. **4**

(b) Discuss the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\operatorname{cosec}\left(\frac{a}{n}\right)}$, where

$\alpha > 0$. **4**

9. (a) Test the convergence of $\sum_{n=1}^{\infty} \frac{\sin nx}{n^p}$. $p > 0$. **4**

(b) Show that $\prod_{n=0}^{\infty}\left[1+\left(\frac{1}{2}\right)^{2n}\right]$ converges to 2. **4**