Roll No. .... Total Pages : 4

GSE/M-21 **1474**

VECTOR CALCULUS

Paper–BM-123

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Evaluate $\hat{i}.(\hat{j} \times \hat{k}) + (\hat{i} \times \hat{k}).\hat{j}$. 2

(b) If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} = z\hat{k}$ prove that $\nabla f(r) \times \vec{r} = \vec{0}$. 2

(c) Let u, v, w be orthogonal co-ordinates, prove that

$$\hat{e}_1 = \hat{\mathrm{E}}_1, \hat{e}_2 = \hat{\mathrm{E}}_2, \hat{e}_3 = \hat{\mathrm{E}}_3.$$ 2

(d) If $\vec{r} = 2t\hat{i} + 3t^2\hat{j} - t^3\hat{k}$, evaluate $\int_1^2 \left(\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2} \right) dt$. 2

SECTION–I

2. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors, such that $\vec{b} \times (\vec{c} \times \vec{a}) = \frac{1}{2}\vec{c}$, find angles which $\vec{b}$ makes with $\vec{c}$ and $\vec{a}$, $\hat{i}$ and $\vec{a}$ being non-parallel. 4

(b) Prove that $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}) = [\vec{a}\,\vec{b}\,\vec{c}]\,\vec{c}$ and hence deduce that $[\vec{b}\times\vec{c}\;\; \vec{c}\times\vec{a}\;\; \vec{a}\times\vec{b}\,] = [\vec{a}\,\vec{b}\,\vec{c}]^2$. 4

3. (a) Show that $[\vec{a}+\vec{b}\;\; \vec{b}+\vec{c}\;\; \vec{c}+\vec{a}\,] = 2[\vec{a}\,\vec{b}\,\vec{c}]$. 4

(b) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have a constant magnitude is $\vec{f}\cdot\dfrac{d\vec{f}}{dt} = 0$. 4

SECTION–II

4. (a) Find the directional derivative of

$$f(x, y, z) = xy + yz + zx$$

in the direction of the vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ at the point (3, 1, 2). 4

(b) Show that $r^n\vec{r}$ is irrotational, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$. 4

5. (a) Explain geometrical interpretation of grad *d*. 4

(b) Prove that $\nabla^2 f(r) = \dfrac{2}{r}f'(r) + f''(r)$. 4

SECTION–III

6. (a) Express the vector field $2y\hat{i} - z\hat{j} + 3x\hat{k}$ in spherical polar co-ordinates. 4

(b) Prove that spherical coordinate system is self-reciprocal. 4

7. (a) Express $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cyclindrical coordinates.

(b) Prove that $u = xy$, $v = \frac{x^2 + y^2}{2}$, $w = z$ are not orthogonal. 4

SECTION–IV

8. (a) Evaluate by Green's theorem

$\oint_C (\cos x \ \sin y - xy)dx + \sin x \cos y \, dy$, where C is the circle $x^2 + y^2 = 1$. 4

(b) Evaluate by Stocke's theorem $\oint_C (e^x \, dx + 2ydy - sz)$ where C is the curve $x^2 + y^2 = 4$, $z = 2$. 4

9. (a) Evaluate $\iint_S (x^3 dydz + y^3 dz\, dx + z^3 dxdy)$ over the surface S of a cube bounded by the coordinate planes and the planes $x = y = z = a$. 4

(b) Show that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C xdy - ydx$. Hence find the area of the ellipse $x = a \cos \theta$, $y = b \sin \theta$. 4