Roll No. ..... Total Pages : 4

GSE/M-21 **1473**

MATHEMATICS
(Ordinary Differential Equation)
Paper–BM-122

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Define exact differential equation. 1

(b) Find the integrating factor of the differential equation $a(ndy + 2ydx) = nydy$. 2

(c) Define Clairaut's equation. 1

(d) Solve the differential equation $\frac{d^2y}{dx^2} + y = n$. 2

(e) Solve the equation $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{yz}$. 2

SECTION–I

2. (a) Solve the differential equation

$$(1 + e^{x/y})dx + e^{x/y}\left[1 - \frac{x}{y}\right]dy = 0.$$ 4

(b) Solve $(x^2 + y^2 + 2x)dx + 2ydy = 0$. 4

3. (a) Solve the differential equations $y = px + \sqrt{a^2 p^2 + b^2}$. 4

(b) Solve the differential equation

$\sin px \cos y = \cos px \sin y + p$ and obtain the singular equation. 4

SECTION–II

4. (a) Find the orthogonal trajectories of the family of co-axal circles $x^2 + y^2 + 2gx + c = 0$, where g is a parameter and c is a constant. 4

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dn} + 4y = n^2 + e^x + \cos 2n. \quad 4$$

5. (a) Solve the differential equation $\frac{d^2y}{dn^2} + 4y = n \sin n$. 4

(b) Solve the differential equation

$$x^2 \frac{d^2y}{dn^2} + 4x \frac{dy}{dn} + 2y = n + \sin n. \quad 4$$

SECTION–III

6. (a) Solve the differential equation

$$x^2 \frac{d^2y}{dn^2} - (x^2 + 2x) \frac{dy}{dn} + (x + 2)y = x^3 e^x. \quad 4$$

(b) Solve $\frac{d^2y}{dx^2} - \tan x \frac{dy}{dx} + 5y = 0$, by removing the first derivation. 4

7. (a) Solve the differential equation

$$x^4 \frac{d^2y}{dx^2} + 2x^3 \frac{dy}{dn} + n^2 y = 0.$$ 4

(b) Solve $n^2 \frac{d^2y}{dn^2} - 2n(1+n)\frac{dy}{dn} + 2(1+n)y = n^3$, by variation of parameters method. 4

SECTION–IV

8. (a) Solve the following simultaneous equations

$$\frac{dn}{dt} - 7n + y = 0$$

$$\frac{dy}{dt} - 2n - 5y = 0.$$ 4

(b) Solve the equations $\frac{dn}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (y+n)^2}$. 4

9. (a) Solve the differential equation

$$(yz + 2x)dn + (zx - 2z)dy + (ny - 2y)dz = 0. \qquad 4$$

(b) Solve the simultaneous equations

$$\frac{xdn}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}. \qquad 4$$
