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MDQ/J-21 5513

BOUNDARY VALUE PROBLEMS

MM-510 (ii)

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section and the compulsory question.

Section I

1. (a) Convert the initial value problem : **8**

$$\frac{d}{ds}\left(p\frac{dy}{ds}\right)+qy=\mathrm{F}(s)$$

$$y(a)=0,\ y'(a)=0$$

into an integral equation.

(b) Solve : **8**

$$y''+y=\mathrm{F}(s)$$

$$y(0)=1,\ y'(0)=-1$$

2. (a) Define $G_M(s;\ t)$ with four properties. **8**

(b) Find the boundary value problem that is equivalent to the integral equation : **8**

$$y(s)=\lambda\int_{-1}^{1}\left(1-|s-t|\right)y(t)dt$$

Section II

3. (a) Transform the exterior Neumann problem :

$$\nabla^2 u_e = 0,\ x \in \mathrm{Re},\ \left.\frac{\partial u_e}{\partial n}\right|_S = f,\ u_e\big|_\infty = 0$$

into an integral equation. **8**

(b) Obtain the electrostatic potential due to a thin circular disk. **8**

4. Solve the Helmholtz equation :

$$\left(\nabla^2 - k^2\right)u = -4\pi\rho$$

with $u\big|_s = \tau$ and $\left.\frac{\partial u}{\partial n}\right|_s = \sigma$ and give properties of three potentials obtain so. **8+8**

Section III

5. (a) Solve the inhomogeneous integral equation : **8**

$$g(s) = 1 - \int_0^s (s-t)\, g(t)\, dt$$

by Laplace transform.

(b) Define finite first form Hilbert transform pair and obtain a fourth form of it. **8**

6. Explain the method of three-part mixed boundary value problems. **16**

Section IV

7. Discuss Perturbation method and its application to electrostatics. **16**

8. Formulate the problem of rotary oscillations in Stokes flow and solve it. **16**

Compulsory Question

9. (i) What do you mean by mixed boundary value problem ? **2**

(ii) Give shifting property of Dirac-Delta function. **2**

(iii) Solve the integral equation : **2**

$$\sin s = \frac{1}{\pi}\int_{-\infty}^{*\infty}\left[\frac{g(t)}{(t-s)}\right]dt$$

(iv) Prove that : **2**

$$\Omega(\mathrm{P}) = \frac{\mathrm{K}(\mathrm{P})}{[1-\mathrm{K}(\mathrm{P})]}$$

(v) Prove that Green's function appearing in the equation :

$$-\nabla^2 \mathrm{G} = \delta(x-\xi),\ \mathrm{G}|_{\mathrm{S}} = 0$$

is symmetric. **2**

(vi) Give two properties of the Newtonian potential : **2**

$$\int_{\mathrm{R}} \frac{\rho}{r}\, d\mathrm{V}$$

(vii) Prove that : **2**

$$\frac{d}{dx}\left[\mathrm{H}(x)\right] = \delta x$$

(viii) Define self-adjoint initial value problem. **2**