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MDE/J-21 **5506**

GENERAL MEASURE AND INTEGRATION THEORY

MM-507

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **9** is compulsory.

Section I

1. (a) State and prove Unique Extension Theorem. **8**

(b) Show that a measure on a ring is countably subadditive. Is it conditionally continuous from above ? Justify. **8**

2. (a) If f and g are measurable functions defined on X and C is any real number, show that :

$$A = \{x : f(x) < g(x) + c\}$$

is locally measurable. **8**

(b) If $Q : R \to R$ is continuous, show that Q is Borel measurable. **8**

Section II

3. State and prove Egoroff's theorem. **16**

4. (a) If $0 \le f \le g$, where g is integrable and f is measurable, show that f is also integrable and $\int f d\mu \le \int g d\mu$. **8**

(b) If f is a measurable function, show that the following are equivalent : **8**

(i) f is integrable

(ii) $|f|$ is integrable

(iii) f^+ and f^- are integrable.

Section III

5. If h is integrable with respect to π, show that both integrals of h exist, and $\int\int h\, d\nu\, d\mu = \int\int h\, d\mu\, d\nu = \int h\, d\pi$. **16**

6. (a) State and prove Lebesgue decomposition theorem. **8**

(b) If f is integrable with respect to μ, show that μ_f is AC with respect to μ. State clearly the results used by you. **8**

Section IV

7. (a) If C is compact G_s, show that there exists a sequence f_n in $\mathcal{L}$ such that $f_n \downarrow \chi_c$. **8**

(b) "Every Baire measure is regular." Prove or disprove this statement. **8**

8. (a) Show that every function in $\mathcal{L}$ is integrable with respect to any Baire measure or any Borel measure. **8**

(b) State and prove Riesz-Markoff theorem. **8**

(Compulsory Question)

9. (a) Show that an additive positive set function $\mathcal{V}$ defined on a ring is monotone.

(b) Give an example to show that $|f|$ can be measurable without f being measurable.

(c) State (only) Arzela Young theorem.

(d) Give an example to show that convergence in measure does not imply convergence in mean.

(e) State (only) Radon Nikodym theorem.

(f) What do you mean by :

(i) Measurable rectangle

(ii) Cartesian product of two measurable spaces ?

(g) If f is a continuous real valued function on X and C is real number, show that $A = \{x : f(x) \geq c\}$ is a closed G_{δ}.

(h) Define a Borel set and a Borel function. **8×2=16**