Roll No. .... Total Pages : 2

GSQ/M21 1743

REAL AND COMPLEX ANALYSIS

Paper–BM-361

Time allowed : 3 Hours **Maximum Marks : 40**

Note : Attempt **five** questions in all, Question No. **1** is compulsory. Selecting **one** question from each unit. All questions carry equal marks.

Compulsory Question

1. (i) Find the coefficient of magnification and angle of rotation at $z = 2 + i$ for the conformal transformation $w = z^2$. 2

(ii) Show that the function $u(x, y) = \frac{1}{2}\log(x^2 + y^2)$ is harmonic. 2

(iii) Evaluate : $\int_0^{\infty} \sqrt{x}\, e^{-x^3}\, dx$ 2

(iv) Define Fourier Series for odd functions. 2

UNIT-I

2. (i) Show that the function $u = x^2 + y^2 + z^2$, $v = x + y + z$, $w = xy + yz + zx$ are not functionally independent of each other. Also find the relation between them. 4

(ii) Show that : 4

$$\int_0^{\infty} \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\, dx = 2B(m, n)$$

3. (i) Change the order of integration of the following integral and hence evaluate : 4

$$\int_0^{a} \int_{y^2/a}^{y} \frac{y}{(a-x)\sqrt{ax - y^2}}\, dx\, dy$$

(ii) Evaluate $\iiint z(x^2 + y^2 + z^2)\, dxdydz$ through the volume of the cylinder $x^2 + y^2 = 4$ intercepted by the planes $z = 0$ and $z = 2$. 4

UNIT-II

4. (i) If the Fourier Series for $f(x)$ converges uniformly in $(c, c + 2l)$, then prove that : 4

$$\int_c^{c+2l} [f(x)]^2\, dx = l\left[\frac{1}{2} a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)\right]$$

(ii) Obtain the Fourier Series expansion for the function $f(x) = x + x^2$ in $[-\pi, \pi]$. 4

5. (i) Find the Fourier expansion of the function $f(x)$ with period 2π defined as : 4

$$f(x) = \begin{cases} -1 & , \text{for } -\pi < x < 0 \\ 1 & , \text{for } 0 \leq x \leq \pi \end{cases}$$

(ii) Express $f(x) = x$ as half range cosine series in $0 < x < 2$. 4

UNIT-III

6. (i) Find the stereographic projection of the point $z = x + iy$ of extended complex plane on the sphere of radius 1 and centre (0, 0, 0) in R^3. 4

(ii) Show that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at origin. 4

7. (i) Show that the function $f(z) = \sqrt{|xy|}$, $z = x + y$ is not analytic at the origin, although the Cauchy-Hiemann equations are satisfied at that point. 4

(ii) Prove that $u = y^3 - 3x^2y$ is a harmonic function and find the corresponding analytic function. 4

UNIT-IV

8. (i) What is the region of the w-plane into which the rectangular region in the z-plane bounded by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 2$, is mapped under the transformation $w = z + (2 - i)$. 4

(ii) Find the fixed points and normal form of the Mobins transformation : 4

$$w = \frac{z}{z-4}$$

9. (i) Find the billnear transformation which maps the joints $z = 1, i, -1$ onto $w = i, 0, -i$. Also, find the image of $|z| < 1$. 4

(ii) Prove that the image of $|z + 2i| = 5$ under the transformation

$$f(z) = \frac{1}{z} \text{ is } u^2 + v^2 = \frac{1}{21}(1 - 4v).$$ 4