Roll No. .... **Total Pages : 04**

GSM/M-21 **1581**

MATHEMATICS

Special Functions and Integral Transforms

Paper : II

BM-242

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Write the Bessel function $J_0(x)$ in the form of series. **1½**

(b) Verify that Legendre polynomial $P_3(x) = \frac{x}{2}(5x^2 - 3)$ satisfies the Legendre's equation with the parameter n is equal to 3. **1½**

(c) Evaluate $L\left[\frac{e^{-t}\sin t}{t}\right]$. **1½**

(d) Find Fourier transform of $f(x)=\begin{cases}\frac{1}{2}\in, & \text{for}\ |x|\le\in \\ 0, & \text{otherwise}\end{cases}$.

1½

Section I

2. (a) Solve the differential equation in power series :

$$\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$$ **2½**

(b) Solve $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=0$ in series about $x = 0$. **2½**

3. (a) Find the solution of $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+\frac{1}{4}y=0$ in terms of Bessel's function. **2½**

(b) Show that :

$$J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x).$$ **2½**

Section II

4. (a) Show that :

$$(n+1)P_{n+1}(x)+n\ P_{n-1}(x)=(2n+1)x\ P_n(x).$$ **2½**

(b) Prove that :

$$\int_{-1}^{1} x \, P_n(x) \, P_{n-1}(x) = \frac{2n}{4n^2 - 1}.$$ **2½**

5. (a) Show that $H_n(-x) = (-1)^n H_n(x)$. **2½**

(b) Evaluate $\int_{-\infty}^{\infty} x e^{-x^2} H_m(x) H_n(x) \, dx$. **2½**

Section III

6. (a) Find the Laplace transform of the function :

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 3, & t > 2 \end{cases}.$$ **2½**

(b) Using Convolution theorem, evaluate :

$$L^{-1}\left[\frac{1}{s^2\left(s^2 + a^2\right)}\right].$$ **2½**

7. (a) Solve the integral equation :

$$f(t) = 1 + \int_0^t f(u) \sin(t - u) \, du$$

and verify your solution. **2½**

(b) Solve the following equation by using Laplace transform : $\frac{d^2y}{dt^2} + y = 6\cos 2t$, where $y'(0) = 1$, $y(0) = 3$. **2½**

Section IV

8. (a) Find the sine transform of :

$$f(x)=\begin{cases}\sin x, & 0<x<a\\ 0, & x>a\end{cases}.$$ **2½**

(b) Using Parseval's identity, show that :

$$\int_0^\infty \frac{x^2dx}{\left(x^2+1\right)^2}=\frac{\pi}{4}.$$ **2½**

9. (a) Solve $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$, if $u(0,t)=0$, $u(x,0)=e^{-x}$, $x>0$, $u(x,t)$ is bounded when $x>0$, $t>0$. **2½**

(b) Solve the integral equation :

$$\int_0^\infty f(x)\cos sx\,dx=\begin{cases}1-s, & \text{for } 0\le s\le\\ 0, & \text{for } s>1\end{cases}.$$ **2½**

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