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GSE/M-21 **1450**

MATHEMATICS

(Vector Calculus)

Paper–BM-123

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) Find the volume of parallelopiped whose edges are represented by $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$. 1

(b) Interpret the symbol $\vec{a}\,.\,\nabla$. 1

(c) Evaluate $\frac{d}{dt}\left[\vec{r}\ \frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\right]$. 1

(d) Show that $\iint_S \hat{n}\, ds = 0$ for any closed surface S. 1

(e) Find the unit tangent vector at $t = 2$ on the curve $x = t^2 - 1$, $y = 4t - 3$, $Z = 2t^2 - 6t$. 1

SECTION–I

2. (a) Show that $\vec{a} \times (\vec{b} \times \vec{c}),\ \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 2

(b) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are perpendicular to each other, then prove $[\vec{a}\,, \vec{b}\,, \vec{c}\,] = a^2 b^2 c^2$. 2½

3. (a) If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \dfrac{1}{2} \vec{b}$, find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$ where $\vec{b}$ and $\vec{c}$ non-parallel. 2½

(b) Evaluate $\dfrac{d}{dt}\left[\vec{r} \times \left\{\dfrac{d\vec{r}}{dt} \times \dfrac{d^2\vec{r}}{dt^2}\right\}\right]$. 2

SECTION–II

4. (a) Find the directional derivatives of

$$f(x, y, z) = xy + yz + zx$$

in the direction of the vector $2\hat{i} + 3\hat{j} + 3\hat{k}$ at the point (3, 1, 2). 2

(b) If $d = x^2 y^3 z^4$, then find div (grad ϕ) i.e., $\nabla \cdot (\nabla\phi)$. 2½

5. (a) Show that the function $\dfrac{1}{r}$,

where $r = |\vec{r}\,| = \sqrt{x^2 + y^2 + z^2}$

is harmonic function provided $r \neq 0$. 2½

(b) Evaluate $\nabla \cdot (\vec{r} \times \vec{a})$, where $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 2

SECTION–III

6. (a) Let $u = xy$, $v = \frac{x^2 + y^2}{2}$, $w = z$. Show that u, v, w are not orthogonal. 2

(b) If δ, ϕ, z are cylindrical coordinates show that $\nabla \phi$ and $\nabla \log \delta$ are solenoidal. 2½

7. (a) If (r, θ, ϕ) are spherical coordinates show that $\nabla \phi = \nabla \times (r \cos \theta \nabla \theta)$. 2

(b) Find the volume element dv in

(i) cylindrical.

(ii) spherical polar coordinates. 2½

SECTION–IV

8. (a) Evaluate $\iint_S \vec{f} \cdot \hat{n}\, ds$, where $\vec{f} = 12x^2 y\hat{i} - 3yz\hat{j} + 2z\hat{k}$ and S is the surface of the plane $x + y + z = 1$ included in the first octant. 2½

(b) Find the work done in moving a particle in a force field $\vec{f} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the line joining the point (0, 0, 0) and (2, 1, 3). 2

9. (a) State and prove Stokes theorem. 2

(b) Evaluate by Green's theorem

$$\oint_C (x^2 - \cos h\ y)\, dx + (y + \sin x)\, dy$$

where C is the rectangle with vertices

(0, 0), (4, 0), (4, 1), (0, 1). 2½