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GSE/M-21 **1449**

ORDINARY DIFFERENTIAL EQUATION

Paper–Math-BM-122

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) What do you mean by general solution of a differential equation. 1

(b) Define Clairaut's equation. 1

(c) Solve $p = \tan(px - y)$. 1

(d) Determine the complementary function of the differential equation $(D^3 + 1)y = 3 + 5e^x$. 1

(e) Write the auxialiary equation of the simultaneous differential equation

$$\frac{dx}{dt} + y = \sin t;\ \frac{dy}{dt} + x = \cos t.$$ 2

SECTION–I

2. (a) Solve the differential equation

$$\frac{2x}{y^3}\,dx + \left[\frac{y^2 - 3x^2}{y^4}\right]dy = 0.$$ 2½

(b) Solve the differential equation

$$(x^2y^2 + xy + 1)y\, dx + (x^2y^2 - xy + 1)x\, dy = 0. \qquad 2½$$

3. (a) Solve the differential equation

$$p^3 - p(x^2 + xy + y^2) + xy\,(x + y) = 0. \qquad 2½$$

(b) Reduce the differential equation $(px - y)\,(x - py) = 2p$ to Clairaut's form by substitution $x^2 = u$ and $y^2 = v$ and find its complete primitive and its singular solution; if any. 2½

SECTION–II

4. (a) Find the orthogonal trajectories of the $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter. 2½

(b) Solve the differential equation

$$\frac{d^2 y}{dx^2} + 4y = e^x + \sin 3x + x^2. \qquad 2½$$

5. (a) Solve the differential equation

$$\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2 e^{2x} \sin 2x. \qquad 2½$$

(b) Solve the differential equation

$$(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4\cos \log\,(1 + x). \qquad 2½$$

SECTION–III

6. (a) Solve the differential equation

$$(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x(1-x^2)^{3/2}. \qquad 2½$$

(b) Solve the equation by removing the first derivative :

$$\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+\left[n^2+\frac{2}{x^2}\right]y=0. \qquad 2½$$

7. (a) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2}+n^2y=\sec nx. \qquad 2½$$

(b) Solve the equation by using the method of undetermined cofficients

$$(D^2+1)y=2e^x+\cos x. \qquad 2½$$

SECTION–IV

8. (a) Solve the simultaneous equation

$$\frac{d^2x}{dt^2}-3x-4y=0 \text{ and } \frac{d^2y}{dt^2}+x+y=0. \qquad 2½$$

(b) $$\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}. \qquad 2½$$

9. (a) Solve :

$$\frac{dx}{1} = \frac{dy}{-2} = \frac{dz}{3x^2 \sin(y + 2x)}.$$ 2½

(b) Solve the differential equation

$$(y^2 + z^2 - x^2)dx - 2xydy - 2xzdz = 0.$$ 2½