Roll No. .... Total Pages : 3

GSM/M-20 **1613**

Sequences and Series

Paper - BM-241

Time allowed : 3 Hours *Maximum Marks : 40*

Note : Attempt five questions in all, selecting one questions from each Section. Question No. 1 is compulsory.

Compulsory Question

1. (i) Give example of a set S, which is infinite and bounded. 1

   (ii) Find interior points of the set S=(1,2) U{3,4,5}. 1

   (iii) Define compact set. 1

   (iv) State Squeeze principle 1

   (v) Check whether <4, 1, 9, 19,...> is a subsequence of <n> or not. 1

   (vi) State Leibnitz test on Alternating series. 1

   (vii) Show that infinite product $\prod_{n=1}^{\infty}(n+\frac{1}{n})$ is divergent. 1

   (viii) State Dirichlet test for arbitrary series. 1

SECTION-I

2. (a) Prove that Q (set of rationals) is not complete ordered field. 4

   (b) Define closure of a set and prove that : 4

   $\overline{(A\cup B)}=\overline{A}\cup\overline{B}$

3. (i) Prove that A° is an open set. 4

(ii) State and prove Bolzano-Weierstrass theorem. 4

SECTION-II

4. (i) State and prove Cauchy's first theorem on limits. 4

(ii) Give an example of a sequence $<a_n>$ which is not a bounded but for which $<\frac{a_n}{n}>$ is a null sequence. 4

5. (i) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ is convergent to $\frac{1}{4}$ 4

(ii) If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} an = 0$. Is converse true ? Justify your answer. 4

SECTION-III

6. (a) State and prove Gauss Test for infinite series. 4

(b) Test the convergence of 4

$$\frac{a+x}{1} + \frac{(a+2x)^2}{2l.} + \frac{(a+3x)^3}{3l.} + \ldots$$

7. (a) Examine the convergence $\sum_{n=1}^{\infty} (n + \frac{1}{n})^n x^n$, $(x>0)$. 4

(b) Test the convergence of $\sum_{n=1}^{\infty} ne^{-n^2}$ 4

SECTION-IV

8. (i) State and prove Leibnitez's test for convergence of an Alternating series. 4

(ii) Test the convergence $\sum_{n=1}^{\infty} \frac{\cos nx}{n^p}$, $(p>0)$ 4

9. (i) Show that the Cauchy product of the convergent series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}}$ with itself is not convergent. 4

(ii) Show that the infinite product 4

$(1+\frac{1}{2})(1-\frac{1}{3})(1+\frac{1}{4})(1-\frac{1}{5})$ ... congerges to 1.