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GSE/M-20 **1449**

MATHEMATICS

(Ordinary Differential Equations)

Paper : BM-122

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) Write the solution of exact differential equation $M\,dx + N\,dy = 0$. 1

(b) Solve the equation $(y - px)^2 = 1 + p^2$. 1½

(c) Find the solution of equation $\frac{d^4y}{dx^4} - a^4x = 0$. 1½

(d) Write the condition if e^x is a particular solution of

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R.$$ 1

(e) Define Total differential equation. 1

SECTION–I

2. (a) Solve the equation

$(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0.$ 2½

(b) Solve the equation

$$\left(y + \frac{y^3}{3} + \frac{x^2}{2}\right)dx + \frac{1}{4}(x + xy^2)\,dy = 0.$$ 2½

3. (a) Solve the differential equation $y = -px + x^4p^2$. 2½

(b) Solve the equation $\sin px \cos y = \cos px \sin y + p$, and obtain the singular solution. 2½

SECTION–II

4. (a) Find the orthogonal trajectories of the cardioid $r = a(1 - \cos\theta)$, where a is the parameter. 2

(b) Solve the equation $\dfrac{d^2y}{dx^2} + y = \sec x$. 3

5. (a) Solve the equation $x^2\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = e^x$. 3

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = \sin x \sin 2x.$$ 2

SECTION–III

6. (a) Solve the equation

$$x^2 \frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3. \qquad 2\frac{1}{2}$$

(b) Solve the equation by removing the first derivative :

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = (\sec x)e^x. \qquad 2\frac{1}{2}$$

7. (a) Solve the equation by variation of parameters :

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}. \qquad 2\frac{1}{2}$$

(b) Solve the equation by changing the independent variable :

$$\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - y\sin^2 x = 0. \qquad 2\frac{1}{2}$$

SECTION–IV

8. (a) Solve the simultaneous equations

$$t\frac{dx}{dt} + y = 0$$

$$t\frac{dy}{dt} + x = 0$$

given that $x(1) = 1$, $y(-1) = 0$. 3

(b) Solve the equation

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (y + x)^2}.$$ 2

9. (a) Solve the total differential equation

$$2yz\ dx + zx\ dy - xy(1 + z)\ dz = 0.$$ 2½

(b) Solve the equations :

$$\frac{dx}{dt} + 5x + y = e^t.$$

$$\frac{dy}{dt} - x + 3y = e^{2t}.$$ 2½