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GSE/M-20 **1448**

MATHEMATICS

(Number Theory and Trigonometry)

Paper : BM-121

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section.

Compulsory Question

1. (a) If $a|c$, $b|c$ and $(a, b) = 1$, then $ab|c$. 1½

(b) If a is odd, prove that $a^2 \equiv 1 \pmod 8$. 1

(c) Evaluate $d(630)$. 1½

(d) Prove that $\cosh^{-1} x - \sinh^2 x = 1$. 1½

(e) Prove that $\sinh^{-1} x = -i \sin^{-1} (ix)$. 1½

SECTION–I

2. (a) Find the g.c.d. of 275 and 200, and express it in the form $m.275 + n.200$. 2½

(b) Solve the congruence $15x \equiv 12 \pmod{21}$. 2½

3. (a) Show that $2^{48} \equiv 1 \pmod{105}$. 2½

(b) Find the remainder when 2.28! is divided by 31. 2½

SECTION–II

4. (a) Solve the congruences $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 5 \pmod 2$ simultaneously. 2½]

(b) Show that 2, 4, 6, ..., $2m$ is a CRS (mod m) if m is odd. 2½

5. (a) Find highest power of 7 contained in 1000!. 2½

(b) Show that 3 is a quadratic residue of 23. 2½

SECTION–III

6. (a) If $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$,

$b = a + a^2 + a^4$,

$c = a^3 + a^5 + a^6$,

show that b and c are the roots of the equation

$$x^2 + x + 2 = 0.$$ 2½

(b) Express $\sin^7\theta\cos^2\theta$ as a sum of the series of multiples of θ. 2½

7. (a) If $x + iy = \cos(u + iv)$ show that

$$(1 + x)^2 + y^2 = (\cosh v + \cos u)^2.$$ 2½

(b) If tan y = tan α tanh β and tan z = cot α tanh β, prove that tan $(y + z)$ = sinh 2β cosec 2α. 2½

SECTION–IV

8. (a) If $i^{\alpha+i\beta} = a + ib$ prove that $a^2 + b^2 = e^{-(4n+1)\pi\beta}$. 2½

(b) Solve the equation

$$\tan^{-1}\frac{1}{4} + 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{6} + \tan^{-1}\frac{1}{x} = \frac{\pi}{4}.$$ 2½

9. (a) Separate $\tanh^{-1}(x + iy)$ into real and imaginary parts. 2½

(b) Sum to n terms the series

$\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \ldots..$ 2½