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MDQ/M-20 5513

BOUNDARY VALUE PROBLEMS

MM-510 (Opt. ii)

Time : Three Hours] [Maximum Marks : 80

Note : The candidate is required to attempt *Five* questions in all, selecting *one* question from each Section and the compulsory question.

Section I

1. Transform the boundary value problem :

$$y''(s) + A(s)y' + B(s)y = F(s)$$

$$y(a) = y_0,\ y(b) = y_1,$$

into Fredholm integral equation. **16**

2. (a) Obtain the Green's function for the given problem : **8**

$$y'' = F(s)$$

$$y(0) = y(l) = 0$$

(b) Transform the boundary value problem : **8**

$$-\left(\frac{d^2y}{ds^2} + \lambda y\right) = F(s)$$

$$y'(0) = y'(l) = 0,\ 0 \le s \le l$$

into an integral equation.

Section II

3. (a) Obtain the Newtonian, Single-layer, and Double-layer potentials for the equation : **8**

$$-\nabla^2 u = 4\pi\rho$$

$$u / s = \tau, \left.\frac{\partial u}{\partial n}\right|_s = \sigma$$

(b) Obtain the Poisson integral formula. **8**

4. Solve the problem of Acoustic diffraction of a plane wave by a perfectly soft disk. **16**

Section III

5. (a) Solve the integral equation :

$$s = \int_0^s e^{s-t} g(t)\, dt$$

using transform methods. **8**

(b) Find the resolvent of the integral equation : **8**

$$g(s) = f(s) + \int_0^s e^{s-t} g(t)\, dt$$

6. Explain the method of solution of mixed boundary value problem : **16**

$$\int_0^a k_0(t, s)\, g(t)\, dt = f(\rho), \quad 0 < \rho < a$$

and hence solve :

$$\int_0^a t\phi(t) \int_0^\infty J_1(p\rho) J_1(pt)\, dpdt = \Omega\rho, \quad 0 < \rho < a$$

Section IV

7. Solve the problem :

$$f(\mathrm{P}) = \int_{\mathrm{S}} k(\mathrm{P}, \mathrm{Q})\, g(\mathrm{Q})\, dS, \mathrm{P} \in \mathrm{S}$$

using perturbation techniques. **16**

8. (a) Discuss theory of diffraction. **8**

(b) Solve the problem of the diffraction of a plane wave by a soft sphere. **8**

Compulsory Question

9. (i) Define initial and final value problems. **2**

(ii) Find the Jump discontinuity for the kernel : **2**

$$k(s,t) = \begin{cases} \dfrac{\lambda s}{l}(l-t), & s < t \\ \dfrac{\lambda t}{l}(l-s), & s > t \end{cases}$$

(iii) Give two properties of $\delta(x - x_0)$. **2**

(iv) Give two properties of single-layer potential : **2**

$$u = \int_s \frac{\sigma}{r} ds$$

(v) Solve interior Dirichlet Problem. **2**

(vi) Prove that : **2**

$$\mathrm{G(P)} = \frac{\mathrm{F(P)}}{\mathrm{K(P)}}$$

(vii) Solve : **2**

$$\sin \mathrm{S} = \int_0^{\mathrm{S}} \mathrm{J}_0(\mathrm{S} - t)\, g(t)\, dt$$

(viii) Define finite Hilbert transform pair. **2**