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MDQ/M-20 **5506**

GENERAL MEASURE AND INTEGRATION THEORY

MM507

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section and the compulsory question.

Section I

1. Let μ be a measure on a ring R, and define an extended real valued set function μ^* on H(R) by the formula :

$$\mu^*(A) = \text{GLB}\left\{\sum_{1}^{\infty}\mu(E_n) A \subset \bigcup_{1}^{\infty} E_n, E_n \in R\,(n = 1, 2, \ldots)\right\}$$

Show that :

(i) μ* is monotone,

(ii) μ* is countably subadditive

(iii) μ* extends μ. **4+8+4**

2. (a) If f is measurable and Q : R → R is a Borel measurable function such that Q(0) = 0. Show that the composite function Q∘f is also measurable. **8**

(b) Show that any bounded measurable function can be uniformly approximated by simple functions. **8**

Section II

3. (a) State and prove Arzela-Young theorem. **6**

(b) State and prove Riesz-Weyl theorem. **10**

4. (a) If f_n is a sequence of ISF such that $f_n \downarrow 0$, show that $I(f_n) \downarrow 0$. **8**

(b) If f is integrable, g is measurable, and $f = g$ a.e., show that g is also integrable, and $\int f d\mu = \int g d\mu$. **8**

Section III

5. State and prove Fubini's theorem, giving full details. **16**

6. (a) State and prove Radon-Nikodym theorem for a σ-finite measure space. **12**

(b) If r is a signed measure such that $r \perp \mu$ and $r << \mu$, then show that $r = 0$. **4**

Section IV

7. (a) If C is a compact G_δ, show that there exists a sequence f_n in L such that $f_n \downarrow \chi_C$. State clearly the result used by you in the proof. **8**

(b) If C and D are compact G_δ's, show that C – D is regular. **8**

8. (a) Show that every f in L is a Baire function. **8**

(b) State and prove Riesz-Markoff theorem. **8**

(Compulsory Question)

9. (a) State (only) the Lemma on monotone classes.

(b) State (only) a necessary and sufficient condition for a function $f : X \to R$ be measurable.

(c) State (only) Egoroff theorem.

(d) Give an example to show that convergence in measure does not imply convergence in mean.

(e) State (only) a partial converse to the Fubini theorem.

(f) Define an absolutely continuous set function. Give *one* example.

(g) Let f be a continuous real valued function on X, and C is a real number. Show that $A = \{x : f(x) \geq C\}$ is a closed G_δ.

(h) Define a Borel set and a Borel function. **8×2=16**