Roll No. .... **Total Pages : 04**

MDE/M-20 4669

DIFFERENTIAL EQUATIONS-II

MM-411

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) Is Prüffer transformation a polar transformation? Support your answer with example or counter example. **2**

(b) Is there a sufficient condition for a second order differential equation to be non-oscillatory ? State exactly. **2**

(c) What is the relation between common zeros and linear independence of solutions of a higher order linear differential equation ? State exactly. **2**

(d) Define a center and give its geometrical significance in reference to a linear system. **2**

(e) Define a node and construct a system of which (0, 0) is a node. **2**

(f) Give examples of periodic boundary conditions and non-linear BVP. **2**

(g) Explain limit orbit. **2**

(h) State the necessary and sufficient for a second order linear differential equation to be self-adjoint. **2**

Section I

2. (a) Show that the equation : **8**

$$x' = t^n + x^2$$

is reducible to Bessel's equation. Name the type of the given equation. Find its solution.

(b) Find the equation of the form $y''(x) + r(x)y = 0$ which is equivalent to $\{a(x)y'\}' + b(x)y = 0,\ x \geq 0$ such that both the equations oscillate together. Prove your claim. **8**

3. (a) State and prove Abel's formula. **8**

(b) Given a differential equation :

$$x^2y'' + xy' + (x^2 - n^2)y = 0,\ a > 0$$

For $n = 0$, every interval of the form $[a, a+\pi]$ contains at least one zero of any solution of the given differential equation and prove that if $n > 1/2$, then every interval of the form $[a, a+\pi]$ contains at most one zero of any non-trivial of the given differential equation. **8**

Section II

4. (a) Prove that between two consecutive zeros of one of two real linearly independent solutions of a second order linear differential equation on a finite interval, there is exactly one zero of the other solution. **8**

(b) Let $q(x) \in C(0, \infty)$ and $4q^{+} = \lim_{x\to\infty}, \sup(xf(x)) < 1$ where $\int_x^{\infty} q(s)\, ds = f(s)$. Prove that the equation $y''(x) + q(x)y(x) = 0$ is non-oscillatory. **8**

5. (a) For what values of the characteristic roots of the system :

$$\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy,$$

the critical point (0, 0) will be a spiral point ? Prove your statement. **8**

(b) Determine the type and stability of the critical point (0, 0) of the system :

$$\frac{dx}{dt} = 22x - 107y, \frac{dy}{dt} = 35x + 241y$$

Obtain the second order equation which is equivalent to this system. **8**

Section III

6. (a) Explain Lyapunov's direct method for finding the stability of a non-linear system. **10**

(b) What is a periodic solution of a linear system ? Explain its relation with limit cycles. **6**

7. (a) State and prove Benedixson non-existence theorem. **8**

(b) Determine the stability of the critical point or of the null solution of : **8**

$$\frac{dx}{dt} = 12x + 7y - x^3 - xy^2 + x^4$$
$$\frac{dy}{dt} = 24x - y + x^2 - y^3 - x^2 y$$

Can you determine the type of critical points of this non-linear system ? Justify.

Section IV

8. (a) Solve the boundary value problem : **8**

$$\frac{\partial u}{\partial t} = (x, t) = h^2 \frac{\partial^2 u}{\partial x^2}(x, t), \ 0 < x < 1, t > 0;$$
$$u(x, 0) = u_0(x), 0 < x < 1; u(0, t) = u(1, t) = 0, t > 0$$

(b) Prove the orthogonality of characteristic functions, corresponding to different values of a Sturm Liouville Boundary Value Problem. **8**

9. (a) Define a singular linear BVP and give one example. Prove that Green's function is symmetric. **8**

(b) Solve the BVP : **8**

$$x'' = \sin(\pi t); x(0) + x(1) = 0, \ x'(0) + x'(1) = 0$$

using Green's function approach.