Roll No. .... **Total Pages : 04**

MDE/M-20 **4668**

COMPLEX ANALYSIS-II

MM-410

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting at least *one* question from each Section and the compulsory question. All questions carry equal marks.

Section I

1. (a) State and prove Hurwitz's theorem.

(b) Show that :

$$\cos \pi z = \prod_{n=1}^{\infty}\left[1-\frac{4z^2}{(2n-1)^2}\right]$$

2. (a) State and prove Riemann mapping theorem.

(b) If Re(z) > 0, prove that :

$$\Gamma(z) = \int_0^{\infty} e^{-t}t^{z-1}dt .$$

Section II

3. (a) Prove that :

$$\zeta^2(z) = \sum_{n=1}^{\infty} \frac{d(n)}{n^z}$$

for Re $z > 1$, where $d(n)$ is the number of divisors of n.

(b) State and prove Mittag-Leffler's theorem.

4. (a) Show that the function :

$$f(z) = 1 + z + z^2 + z^3 + ... + z^n + ...$$

can be obtained outside the circle of convergence of the power series.

(b) State and prove Schwarz's reflection principle.

Section III

5. (a) Show that Poisson Kernel Pr(θ) satisfies the following properties :

(i) $\frac{1}{2\pi}\int_{-\infty}^{\infty} P_r(\theta)\, d\theta = 1$

(ii) $P_r(\theta) > 0$ for all θ, Pr(–θ) = Pr(θ) and P_r is periodic in θ with period 2π.

(b) State and prove Hornack's theorem for harmonic function.

6. If $f(z)$ is analytic within and on the circle r such that $|z| = \mathrm{R}$ and if $f(z)$ has zeros at the points $a_i \neq 0(i = 1, 2, ..., m)$ and poles at $b_j \neq 0(j = 1, 2, ...n)$ inside r, multiple zeros and poles being repeated, then show that :

$$\frac{1}{2\pi}\int_0^{2\pi} \log\left|f\left(\mathrm{Re}^{i\theta}\right)\right| d\theta = \log|f(0)| + \sum_{i=1}^{m} \log\frac{\mathrm{R}}{|a_i|} - \sum_{i=1}^{n} \log\frac{\mathrm{R}}{|b_j|}$$

Section IV

7. (a) State and prove Borel's theorem.

(b) State and prove Hadamard's factorization theorem.

8. (a) State and prove Schottky's theroem.

(b) State and prove $\frac{1}{4}$ theorem.

Compulsory Question

9. (i) State Montel's theorem.

(ii) Find $\Gamma(1), \Gamma(2)$ and $\Gamma\left(\frac{1}{2}\right)$.

(iii) Define Riemann zeta function.

(iv) Define function element.

(v) State Harnack's inequality.

(vi) Construct the canonical product associated with sequence of negative integers.

(vii) Find the order of sin z.

(viii) State Bieberbach conjecture