Roll No. .... **Total Pages : 03**

MDE/M-20 **4666**

REAL ANALYSIS-II

MM-408

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section and the compulsory question.

Section I

1. (a) Show that the outer measure of an interval is its length. **10**

(b) Show that the interval (a, ∞) is measurable. **6**

2. (a) Give an example of a set which is not Lebesgue measurable. **8**

(b) Let C be a constant and f and g two measurable real-valued functions defined on the same domain. Show that the functions $f+c$, cf, $f+g$, $g-f$, and fg are also measurable. **8**

Section II

3. (a) State and prove Lusin's theorem. **10**

(b) State and prove F. Riesz theorem. **6**

4. (a) Let f be a bounded function defined on $[a, b]$. If f is Riemann integrable on $[a, b]$, show that f is measurable and $R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$. **8**

(b) State and prove Bounded Convergence theorem. **8**

Section III

5. (a) State and prove Fatou's lemma. Also show that strict inequality may occur in Fatou's lemma. **8**

(b) If f and g are integrable over E, show that $f + g$ is integrable over E, and $\int_E f + g = \int_E f + \int_E g$. **8**

6. Let f be an increasing real-valued function on the interval $[a, b]$. Show that f is differentiable almost everywhere, the derivative f' is measurable and

$$\int_a^b f(x)\,dx \le f(b) - f(a).$$ **16**

Section IV

7. (a) If f is integrable on $[a, b]$ and $\int_a^x f(t)\,dt = 0$ for all $x \in [a, b]$, show that $f(t) = 0$ a.e. in $[a, b]$. **8**

(b) Show that a function F is an indefinite integral if and only if it is absolutely continuous. **8**

8. State and prove Riesz representation theorem for bounded linear functionals on L^p spaces. **16**

Compulsory Question

9. (a) Show that the set [1, 2] is uncountable.

(b) Give an example of :

(i) G_δ set

(ii) F_σ set.

(c) Give an example of a function f such that f is not measurable but $|f|$ is measurable.

(d) State (only) Lebesgue theorem regarding points of discontinuities of Riemann integrable functions.

(e) State (only) Vitali's covering lemma.

(f) Give an example of a function to show that a continuous function need not be of bounded variation.

(g) State (only) Jensen's inequality.

(h) Show that every bounded function on E is in $L^\infty(E)$.

8×2=16