Roll No. ..... **Total Pages : 05**

GSQ/M-20 **1723**

MATHEMATICS

BM-363

Dynamics

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) A particle describes an equiangular spiral $r = ae^{2\theta}$ with constant angular velocity. Find its radial acceleration. **1**

(b) The maximum velocity of a body moving with S.H.M. is 2 unit/sec. and its period is $\frac{1}{5}$ sec. What is its amplitude ? **1**

(c) Define Hook's law and modulus of elasticity. **2**

(d) Show that at an apse, the particle moves at right angle to radius vector. **2**

(e) A particle is projected with a velocity of 49 m/sec. in a direction making an angle of 45º with the horizontal. Find (i) Time of flight (ii) Horizontal range. **1**

Unit I

2. (a) Find the expressions for tangential and normal components of acceleration of a particle moving along a plane curve. **2½**

(b) To a passenger in an open car travelling of 20 km/hr; the wind appears to come from a direction 60º to the right and from ahead at 4 km/hr. What is the true direction and velocity of the wind ? **2½**

3. (a) A point moving with S.H.M. has a velocity of 6 cm/sec. when passing through the centre of its path and its period is $\frac{2\pi}{3}$ sec. Find its velocity when it has moved a distance $\frac{3}{2}$ cm from its extreme position. **2½**

(b) A light elastic string of natural length l has one end fixed at a point O and the other end attached to a particle, the weight of which in equilibrium position would extend the string to a length l_1. Show that if the particle be dropped from rest at O, it will come to instantaneous rest at a depth $\sqrt{l_1^2 - l^2}$ below the equilibrium position. **2½**

Unit II

4. (a) A mass of 10 kg falls freely a distance of 10 m from rest and is then brought to rest after penetrating through 1 m in sand. Find the average force exerted by the sand on it. **2½**

(b) A bullet weighing 81 gm and moving at the rate of 200 cm/sec. is fired into a log of wood into which it penetrates 10 cm. If the bullet moving with the same velocity were fired into a similar piece of wood 5 cm thick, with what velocity would it emerge ? Also find the force of resistance, supposing it to be uniform. **2½**

5. (a) A train whose mass is 150 tons has an engine of 230 H.P. Find the greatest uniform speed that can be maintained while ascending an incline of 1 in 80, the resistance being equal to the weight of one ton. **2½**

(b) Prove that a train of weight W tons going up an incline of 1 in m will acquire a velocity $\left(\frac{P}{W}-\frac{1}{m}-\frac{R}{2240}\right)$ gt ft/sec. and kinetic energy $\frac{1}{2}W\left(\frac{P}{W}-\frac{1}{m}-\frac{R}{2240}\right)$ gt^2ft tons after t sec. from rest, P being pull of the engine in tons and R the resistance on the level in pounds per ton. **2½**

Unit III

6. (a) A particle is projected with velocity u from the lowest point and moves along the inside of a smooth vertical circle. Discuss the motion. **2½**

(b) A particle is projected very close to the vertex of a smooth cycloid whose axis is vertical and vertex upwards and is allowed to run down the curve. Discuss its motion. **2½**

7. (a) Find the velocity and direction of projection of shot which passes in a horizontal direction over the top of a wall 64 ft. high and 192 ft. distant from the gun. **2½**

(b) Shots fired simultaneously from the top and the bottom of a vertical cliff with elevation 'a' and 'b' respectively strike an object simultaneously at the same point. Show that if 'a' is the horizontal distance of the object from the cliff, the height of cliff is $a(\tan\alpha - \tan\beta)$. **2½**

Unit IV

8. (a) A particle moving under the influence of a central force, describes a circle through the centre of the force. Prove that the force is attractive and inversely proportional to the fifth power of the distance. **2½**

(b) A particle moves with a central acceleration $\left(\frac{\lambda}{(\text{distance})^3}\right)$. Find the path and distinguish the cases. **2½**

9. (a) If v_1 and v_2 are the maximum and minimum velocities of a planet, then prove that $(1-e)v_1+(1+e)v_2$ for an elliptic path. **2½**

(b) A smooth helix is placed with its axis vertical and a small bead slides down it under gravity. Show that it makes its first revolution from rest in time $2\sqrt{\frac{\pi a}{g\sin\alpha\cos\alpha}}$, where α is the angle of helix. **2½**