GSQ/M-20   **1721**
MATHEMATICS
BM-361
Real and Complex Analysis

Time : Three Hours]   [Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Evaluate $\int_0^3 \dfrac{dx}{\sqrt{3x - x^2}}$.   **2**

   (b) Find the Fourier coefficient b_n for the function $f(x) = |x|$ in $[-\pi, \pi]$.   **1**

   (c) Find the image of the point $2 + 3i$ on the sphere of radius $\dfrac{1}{2}$ and centre $\left(0, 0, \dfrac{1}{2}\right)$.   **2**

   (d) Define Orthogonal System.   **1**

   (e) Find the fixed point and normal form of the bilinear transformation $w = \dfrac{3z - 4}{z - 1}$.   **1**

Section I

2. (a) If $u = x + y + z$, $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$, then show that $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} = 0.$ Also find the relation between u, v, w. **2½**

(b) Prove that : **2½**

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}\,dx = \frac{\overline{|(m)}\ \overline{|(n)}}{a^n(1+a)^m\ \overline{|(m+n)}}$$

3. (a) Evaluate $\iiint\limits_V z^2\,dx\,dy\,dz$, where V is interior of sphere $x^2 + y^2 + z^2 = 1$. **2½**

(b) Evaluate $\displaystyle\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ by changing the order of integration. **2½**

Section II

4. (a) Find the Fourier series expansion of $f(x) = x\sin x$ in $[-\pi, \pi]$. **2½**

(b) Express $f(x) = x$ as a half range sine series in $0 < x < 2$. **2½**

5. (a) Obtain Fourier series for the function : **2½**

$$f(x) = \begin{cases} \pi x & , & 0 \le x \le 1 \\ \pi(2-x) & , & 1 \le x \le 2 \end{cases}$$

(b) Find the Fourier series of x^2 in $[-\pi, \pi]$. Use Parseval's identity to prove that : **2½**

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots\ldots\ldots\ldots\ldots$$

Section III

6. (a) Show that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin. **2½**

(b) Prove that the function $\sinh z$ is analytic and find its derivative. **2½**

7. (a) Prove that $u = x^2 - y^2$ and $v = \dfrac{y}{x^2 + y^2}$ are harmonic functions but u and v are not harmonic conjugates. **2½**

(b) Find the regular function whose imaginary part is $v = e^x \left(x \sin y + y \cos y \right)$. **2½**

Section IV

8. (a) Find the image of the infinite strip $0 < y < \dfrac{1}{2}$ under the transformation $w = \dfrac{1}{z}$ and also draw the graph. **2½**

(b) Find the image of region inside the circle $|z| = 1$ under the mapping which maps the points $z = 1,\ i,\ -1$ onto $w = i,\ 0,\ -i$. **2½**

9. (a) Find the condition that the transformation $w = \dfrac{az+b}{cz+d}$ transforms the unit circle in the w-plane into straight lines in the z-plane. **2½**

(b) Find all the Mobius transformations which map the unit circle $|z| \leq 1$ into the unit circle $|w| \leq 1$. **2½**