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**GSM/M-20** **1580**

# MATHEMATICS

(Special Functions and Integral Transforms)

Paper–BM–242

Time Allowed : 3 Hours] [Maximum Marks : 26

**Note :** Attempt **five** questions in all, selecting at least **one** question from each Unit. Question No. **1** is compulsory.

## Compulsory Question

1. (a) Determine the radius of convergence of the power series

$$\sum_{m=0}^{\infty} \left(\frac{2}{3}\right)^{m} x^{2m}.$$ 1½

(b) Write Legendre's equation. 1½

(c) Find Laplace transform of t $\sin^2$ t. 1½

(d) Find the Fourier cosine transform of the function $e^{-mx}, m > 0$ by using its second derivative 1½

## UNIT–I

2. (a) Find the solution of the following equation in terms of Bessel's function

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0.$$ 2½

(b) Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\,d\theta$ for all integral values of n. 2½

3. (a) Solve the following differential equation about x = 0 :

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + x^2y = 0.$$ 2½

(b) Prove that :

$$\int_0^r x\,J_0(ax)\,dx = \frac{r}{a}J_1(ar).$$ 2½

## UNIT–II

4. (a) Show that

$$P_n(-x) = (-1)^n P_n(x).$$ 2½

(b) Prove that

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\,dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

Hence deduce the value of

$$\int_{0}^{1} x^2\, P_{n+1}(x) P_{n-1}(x)\,dx.$$ 2½

5. (a) Prove that if m< n,

$$\frac{d^m}{dx^m}\{H_n(x)\} = \frac{2^m\, n!}{(n-m)!} H_{n-m}(x).$$ 2½

(b) Prove that $\int_{-\infty}^{\infty} x^2 e^{-x^2} \left[H_n(x)\right]^2 dx$

$$= \sqrt{\pi}\, 2^n\, n!\left(n + \tfrac{1}{2}\right).$$ 2½

## UNIT–III

6. (a) Evaluate $L\left(\frac{1-\cos 2t}{t}\right)$. 2½

(b) Find the inverse Laplace transform of

$$\frac{1}{S(S+2)^3}.$$ 2½

7. (a) Convert the differential equation

$$f''(t) - 1f'(t) + 2f(t) = 4\sin t$$

into integral equation where,

$f(0) = 1, f'(0) = -2.$ 2½

(b) Solve $\dfrac{d^4y}{dt^4} - k^4y = 0$ by transform method, where

$y(0) = 1,\ y'(0) = y''(0) = y''(0) = 0.$ 2½

## UNIT–IV

8. (a) Find the sine transform of

$f(x) = 2x,$ where $0 < x < 4.$ 2½

(b) Using Parseval's identity show that

$$\int_0^\infty \frac{dx}{\left(x^2+1\right)^2} = \frac{\pi}{4}.$$ 2½

9. (a) Solve $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ given that

$u(0,t) = 0,\ u(\pi,t) = 0,\ u(x,0) = 2x$

when $0 < x < \pi,\ t > 0..$ 2½

(b) Solve the integral equation

$$\int_0^\infty f(x)\cos 5x\,dx = e^{-s}.$$ 2½