

Roll No.

Total Pages : 4

GSM/M-20

1579

MATHEMATICS

(Sequences & Series)

Paper-BM-241

Time Allowed : 3 Hours]

[Maximum Marks : 27

Note : Attempt **five** questions in all, selecting at least **one** question from each Unit. Question No. 1 is compulsory.

Compulsory Question

1. (a) Define Interior point of a set. 1
- (b) Define Limit Point of a set. 1
- (c) Give an example of an unbounded set which has no limit point. 1
- (d) State Cauchy's root test. 1
- (e) Define Null sequence. 1

UNIT-I

2. (a) Prove that the interior of set A is the largest open subset of A. 3

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- (b) The intersection of an arbitrary family of closed sets is closed. 2½
3. (a) Prove that derived set of any set is a closed set. 3
- (b) Prove or disprove $A^\circ \cup B^\circ = (A \cup B)^\circ$. 2½

UNIT-II

4. (a) Show that

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{1}{(n+1)^2} + \dots + \frac{1}{(2n)^2} \right) = 0. \quad 3$$

- (b) Examine the convergence of the series

$$\sum_{n=3}^{\infty} x^{\log n}. \quad 2½$$

5. (a) State and prove Cauchy's second theorem on limits. 3

(b) Show that $\lim_{n \rightarrow \infty} \left(\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \dots \cdot \frac{n}{n-1} \right)^{\frac{1}{n}} = 1$. 2½

UNIT-III

6. (a) State and prove Cauchy's root test. 3

(b) Test the convergence of the series

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots, x > 0. \quad 2\frac{1}{2}$$

7. (a) State and prove D'Alembert's ratio test. 3

(b) Discuss the convergence of the series

$$\frac{1}{2} + \left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots \quad 2\frac{1}{2}$$

UNIT-IV

8. (a) Show that

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \cdot a_n \quad (p \geq 0)$$

is convergent if $\sum_{n=1}^{\infty} a_n$ is convergent. 3

(b) The product

$$\prod_{n=1}^{\infty} (1 + a_n)$$

is absolutely convergent if and only

$$\sum_{n=1}^{\infty} a_n$$

is absolutely convergent. 2½

9. (a) Show that the series

$$\frac{\log^2}{2^2} - \frac{\log^3}{3^2} + \frac{\log^4}{4^2} - \dots$$

converges.

3

(b) Show that the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left(1 + \frac{1}{n}\right)^{-n}$$

is convergent.

2½