FERMI DIRAC DIST. LAW:

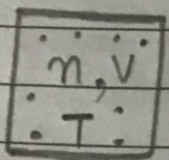
fermi dirac statistical approach is applied to :

1.) indistinguishable particle which obey pauli exclusion principle.

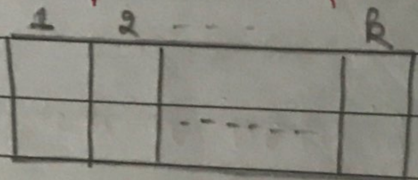
2.) These particles have half Integral spin angular momentum in units of $\frac{h}{2\pi}$

3.) N_{Total} is fixed ; E_{total} is fixed.

Let us consider a system of n fermions obeying F-D statistics with Volume V at temperature T.



That system is further divided into k compartments



such that $n_1, n_2, n_3 \ldots n_k$ are number of particles in each with $u_1, u_2 \ldots u_k$ as the energy respectively.

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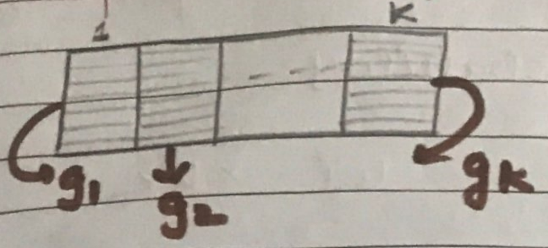
Now, Let $g_1, g_2, g_3 --- g_k$ be the elementary cells in compartments 1, 2, 3 --- K respt.



As each cell can contain either one or zero particles due to pauli exclusion principle, so the number of cells should be more than or equal to number of particles in each compartment. Therefore; $g_i > n_i$

Now, what is the Thermodynamic probability?

Total no. of ways of arrangement $= {}^{g_i}C_{n_i}$.

★ $= \dfrac{g_i!}{g_i!(g_i-n_i)!}$ [As order does not matter here]

★ Here we have used Permutation & Combination theory of Mathematics. We are selecting cells out of g_i cells for n_i particles. Particles are same so order does not matters.: combination principle Rule : ie $nC_r = \dfrac{n!}{r!(n-r)!}$

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Thermodynamic probability of macrostate $(n_1, n_2, \ldots n_k)$ is given by

Total Thermodyn. Probability =

$$W_1 \times W_2 \times \ldots W_i \ldots \times W_k$$

$$W = \prod_{i=1}^{k} \frac{g_i!}{n_i!\,(g_i-n_i)!}$$

Taking log on both sides:

$$\ln W = \sum_{i=1}^{k} \left[\ln[g_i!] - \ln(n_i)! - \ln(g_i-n_i)!\right]$$

Using Stirling formula $\ln n! = n\ln n - n$.

$$= \sum_{i=1}^{k} \left[g_i \ln g_i - n_i \ln n_i - \cancel{g_i} + \cancel{n_i} - (g_i-n_i)\ln(g_i-n_i) + \cancel{(g_i-n_i)}\right]$$

$$= \sum_{i=1}^{k} \left[g_i \ln g_i - n_i \ln n_i - (g_i-n_i)\ln(g_i-n_i)\right] \quad \text{— (1)}$$

Differentiating equ. — (1) keeping g_i as constant.

$$d\ln(W) = \sum_{i=1}^{k} \left[-dn_i \ln n_i - dn_i + dn_i \ln(g_i-n_i) - (-dn_i)\right]$$

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$$= \sum_{i=1}^{k} [\ln(g_i - n_i)\, dn_i - \ln n_i\, dn_i - \cancel{dn_i} + \cancel{dn_i}]$$

$$d\ln(W) = \sum_{i=1}^{k} [\ln(g_i - n_i) - \ln n_i]\, dn_i \quad (1)$$

For Most Probable state $d\ln W = 0$

$$\therefore \sum_{i=1}^{k} [\ln(g_i - n_i) - \ln n_i]\, dn_i = 0.$$

Also using (3) point in the intro i.e.

$$n = n_1 + n_2 + \cdots n_k = \sum_{i=1}^{k} n_i = \text{constant}$$

$$\therefore dn = \sum_{i=1}^{k} dn_i = 0 \quad — (2)$$

AND

$$U = n_1 u_1 + n_2 u_2 + \cdots n_k u_k = \sum_{i=1}^{k} n_i u_i = \text{const}$$

$$\therefore dU = \sum_{i=1}^{k} u_i\, dn_i = 0 \quad — (3)$$

Multiplying eqn. (2) by $-\alpha$ and
eqn. (3) by $-B$
And adding them to (1)
we get;

$$\sum_{i=1}^{k} [\ln(g_i - n_i) - \ln n_i - \alpha - B u_i]\, dn_i = 0$$

$$\ln(g_i - n_i) - \ln n_i - \alpha - B u_i = 0.$$

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$$\ln\left(\frac{g_i - n_i}{n_i}\right) = \alpha + \beta u_i$$

$$\frac{g_i - n_i}{n_i} = e^{\alpha + \beta u_i}$$

$$\frac{g_i}{n_i} - 1 = e^{\alpha + \beta u_i}$$

$$\Rightarrow \boxed{n_i = \frac{g_i}{e^{\alpha + \beta u_i} + 1}}$$ where $\beta = \frac{1}{KT}$

This eqn. represents: fermi dirac distribution law for energy.

$$\boxed{n(u)\,du = \frac{g(u)\,du}{e^{\alpha} e^{u/KT} + 1}}$$

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