Insertion And Removal of Parentheses

Sum of finite number of terms remain same, no matter how the terms are arranged

But this in not true in case of sum of infinite number of terms

Dearrangement of terms of an infinite series may not only change the sum of the series but also change the Convergence, divergence or Oscillation.

Consider the series $\sum_{n=1}^{\infty} (-1)^{n-1} = 1-1+1-1+1-1+\cdots$

This is Oscillating between 0 and 1.
IF paranthesis are inserted

$$\underbrace{(1-1)}_{0} + \underbrace{(1-1)}_{0} + \underbrace{(1-1)}_{0} + \cdots$$

each term of series is zero and hence the series Converges to zero.

A IF we write series as

$$1 - \underbrace{(1-1)}_{0} - \underbrace{(1-1)}_{0} - \cdots$$

then this new series Converges to 1

Hence, we conclude that the insertion of parantheses Can effect the behaviour of the series.

Results of some Theorams Based on (Insertion and Removal of Parentheses)

Theoram ① (Insertion of Paranthesis)

Any series obtained from a convergent series by inserting the paranthesis converges and has the same sum as the Original series

Theoram ② (Removal of Paranthesis)

if the series $\sum_{n=1}^{\infty} b_n$ with Paranthesis converges to S and if $\sum_{n=1}^{\infty} a_n$ obtained from it by the removal of paranthesis also converges, then $\sum_{n=1}^{\infty} a_n$ also converges as

Theoram ③

IF the series $\sum_{n=1}^{\infty} b_n$ with Paranthesis Converges to S and if the sum of the absolute values of the terms in b_n tend to zero as $n \to \infty$, then the seris $\sum_{n=1}^{} a_n$ obtained on removing the parantheses will also Converge to S

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Example ① Show that the series $\left(1-\frac{1}{2}\right)+\left(1-\frac{3}{4}\right)+\left(1-\frac{7}{8}\right)+\cdots$ is convergent, but when the parantheses are removed, it oscillate.

Solution : The given series with parantheses is

$$\sum_{n=1}^{\infty} b_n = \left(1-\frac{1}{2}\right)+\left(1-\frac{3}{4}\right)+\left(1-\frac{7}{8}\right)+\cdots$$

$$= \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$$

which is convergent [$\because$ geometric series with $r=\frac{1}{2}<1$]

and its sum $= \dfrac{\frac{1}{2}}{1-\frac{1}{2}} = 1$ $\left[\because S_\infty = \dfrac{a}{1-r}\right]$

$$b_n = \frac{1}{2^n}+1-1$$

$$= 1-1+\frac{1}{2^n} = 1-\left(1-\frac{1}{2^n}\right)$$

$$= 1-\left(\frac{2^n-1}{2^n}\right)$$

Sum of the absolute values of terms in b_n is

$b_n = |1| + \left|-\left(\frac{2^n-1}{2^n}\right)\right| = 1+\frac{2^n-1}{2^n} = 2-\frac{1}{2^n} \to 2$ as $n\to\infty$

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Sum of absolute values of terms in b_n does not tend to 0

By the result of theorem (3) the series

$$\sum_{n=1}^{\infty} a_n = 1-\frac{1}{2}+1-\frac{3}{4}+1-\frac{7}{8}+\cdots$$

Obtained on removing parentheses in $\sum_{n=1}^{\infty} b_n$ is not convergent.

Now, Find the behaviour of series by partial sum.

$$S_{2n} = \underbrace{1-\frac{1}{2}}+\underbrace{1-\frac{3}{4}}+\underbrace{1-\frac{7}{8}}+\cdots \text{ to } 2n \text{ terms}$$

$$= \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots \text{ to } n \text{ term}$$

$$= \frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots+ n \text{ terms}$$

$$= \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} \qquad \left[\because S_n = \frac{a\left(1-r^n\right)}{1-r}\right]$$

$$\lim_{n\to\infty} S_n = \frac{\frac{1}{2}(1-0)}{\frac{1}{2}} = 1 \qquad \left[\because 0<a<1, \ \lim_{n\to\infty} a^n = 0\right]$$

$$\lim_{n \to \infty} S_{2n+1} = \lim_{n \to \infty} S_{2n} + a_{2n+1} = 1+1 = 2$$

Hence, the series is Oscillatory

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