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B.Sc / BA (III) Sub → Real and Complex Analysis

Important questions :-

Que 1 → If $x = r \sin\theta \cos\phi$, $y = r \sin\theta \sin\phi$, $z = r\cos\theta$, find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$.

Que 2 → Find the Jacobian of u, v, w w.r.t to x, y, z if $u = x+y+z$, $v^3 = yz+zx+xy$, $w^5 = xyz$.

Que 3 → If $u = \frac{x}{y-z}$, $v = \frac{y}{z-x}$, $w = \frac{z}{x-y}$

verify that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$.

Que 4 → If $x = r\cos\theta\cos\phi$, $y = r\sin\theta\sqrt{1-m^2\sin^2\phi}$, $z = r\sin\phi\sqrt{1-n^2\sin^2\theta}$, where $m^2+n^2=1$ then show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = \frac{r^2(m^2\cos^2\phi + n^2\cos^2\theta)}{[(1-m^2\sin^2\phi)(1-n^2\sin^2\theta)]^{1/2}}$$

Que 5 → Prove that the functions

$u = \sin^{-1}x + \sin^{-1}y$, $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

are functionally dependent. Also, find relation b/w them.

Que6 ⇒ Show that $ax^2+2hxy+by^2$ and $Ax^2+2Hxy+By^2$ are independendent unless $\frac{a}{A} = \frac{b}{B} = \frac{h}{H}$.

Que7 ⇒ Show that the f^ns $u = 3x+2y-3$, $v = x-2y+3$ and $w = x(x+2y-3)$ are not independent. Also, find relation b/w them.

Que8 ⇒ (i) Show that $B(m,n) = B(n,m)$

(ii) Show that,

$$B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}\,dx;$$

$m,n>0$

Que9 ⇒ Prove that,

$$B(m,n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!},$$ if m,n are positive integers.

Que10 ⇒ Prove that,

$$\int_a^b (x-a)^m (b-x)^n\,dx = (b-a)^{m+n+1}\,B(m+1,n+1).$$

Que11 ⇒ Prove that, $B(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta$

Que12 ⇒ By putting, $\frac{x}{1-x} = \frac{az}{1-z}$, where a is a suitably selected constant, show that

$$\int_0^1 x^{-1/3}(1-x)^{-2/3}(1+2x)^{-1}\,dx = \frac{1}{9^{1/3}}\,B\left(\frac{2}{3},\frac{1}{3}\right).$$

Que13) Derive relation b/w Gamma and Beta function.

Que14) Prove that,

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}\,dx = \frac{\Gamma(m)\,\Gamma(n)}{a^n(1+a)^m\,\Gamma(m+n)}$$

Que15) State and prove duplication formula.

i.e, $\Gamma(m)\,\Gamma(m) = \dfrac{\sqrt{\pi}}{2^{2m-1}}\,\Gamma(2m)$.

Que16) Prove that, $\Gamma\left(n+\frac{1}{2}\right) = \dfrac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$

Que17) Prove that, $\Gamma(n)\,\Gamma(1-n) = \dfrac{\pi}{\sin n\pi}$,

when $\displaystyle\int_0^\infty \frac{x^{n-1}}{1+x}\,dx = \frac{\pi}{\sin n\pi}$.

Que18) Determine the Fourier coefficients or Euler formulae.

Que19) Find the fourier series expansion of $f(x) = e^x$ in $-\pi < x < \pi$.

Hence, deduce that

$$\frac{\pi}{\sinh\pi} = 1 + 2\sum_{n=1}^{\infty}\frac{(-1)^n}{1+n^2}$$

Que20) Obtain Fourier series for $f(x) = \sqrt{1-\cos x}$ in the interval $(0, 2\pi)$ and, hence find the value of,

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots$$

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Que 21 Find the fourier series for the f^n $f(x) = |\sin x|$; $-\pi < x < \pi$.

Que 22 Find the Fourier series expansion of the f^n f in $(0, 2\pi)$ defined as,

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

Also, find sum of series at $x = \pi$.

Que 23 Find the Fourier series for the function,

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ k, & -1 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

Que 24 Find the half range cosine series of $f(x) = x(\pi - x)$ in $(0, \pi)$.

Que 25 Obtain a Half range cosine series for $f(x) = \begin{cases} kx, & 0 \le x \le \frac{l}{2} \\ k(l-x), & \frac{l}{2} \le x \le l \end{cases}$

Hence, deduce the sum of the series,

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots ?$$

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Que 26 ⇒ Show that the f^n $f(z) = z^2$ is uniformly cnts in the region $|z|<1$.

Que 27 ⇒ Show that the f^n $f(z) = |z|^2$ is cnts everywhere, but nowhere differentiable except at origin.

Que 28 ⇒ Show that $f(z) = \bar{z}$ is nowhere differentiable but cnts everywhere.

Que 29 ⇒ State and prove the necessary condition for the f^n $f(z)$ to be analytic.

Que 30 ⇒ Give an exp to show that C-R eqns are necessary for a f^n to be analytic, not sufficient.

Que 31 ⇒ Prove that an analytic f^n with constant modulus is constant.

Que 32 ⇒ If $f(z)$ is a real valued analytic f^n, then prove that $f(z)$ is constant.

Que 33 ⇒ Prove that real and imaginary parts of an analytic f^n are Harmonic.

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Que 34 ⇒ If $f(z)$ is a Holomorphic f^n, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2.$$

Que 35 ⇒ If $u-v = \dfrac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ and $f(z) = u+iv$ is an analytic f^n of z, then find $f(z)$ in terms of z such that

$$f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}.$$

Que 36 ⇒ Show that $u(x,y) = e^{-x}(x \sin y - y \cos y)$ is Harmonic, and find $v(x,y)$ such that $f(z) = u+iv$ is analytic.