BCA III RD UNIT OF COMPUTER GRAPHICS

2D TRANFORMATION

Transformation means changing some graphics into something else by applying rules. We can have various types of transformations such as translation, scaling up or down, rotation, shearing, etc. When a transformation takes place on a 2D plane, it is called 2D transformation.

Transformations play an important role in computer graphics to reposition the graphics on the screen and change their size or orientation.

Homogenous Coordinates

To perform a sequence of transformation such as translation followed by rotation and scaling, we need to follow a sequential process −

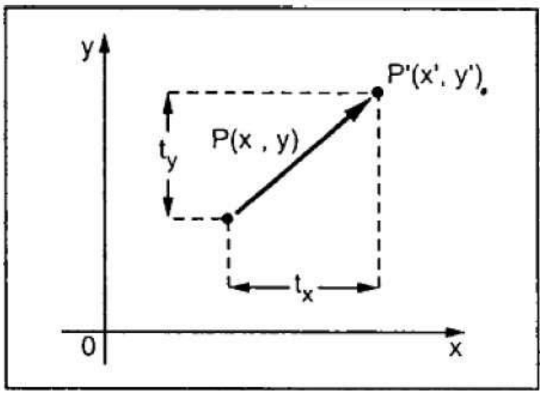
- Translate the coordinates,
- Rotate the translated coordinates, and then
- Scale the rotated coordinates to complete the composite transformation.

To shorten this process, we have to use 3×3 transformation matrix instead of 2×2 transformation matrix. To convert a 2×2 matrix to 3×3 matrix, we have to add an extra dummy coordinate W.

In this way, we can represent the point by 3 numbers instead of 2 numbers, which is called **Homogenous Coordinate** system. In this system, we can represent all the transformation equations in matrix multiplication. Any Cartesian point PX,YX,Y can be converted to homogenous coordinates by P’ (X_h, Y_h, h).

Translation

A translation moves an object to a different position on the screen. You can translate a point in 2D by adding translation coordinate (t_x, t_y) to the original coordinate X,YX,Y to get the new coordinate X′,Y′X′,Y′.



From the above figure, you can write that −

X’ = X + t_x

Y’ = Y + t_y

The pair (t_x, t_y) is called the translation vector or shift vector. The above equations can also be represented using the column vectors.

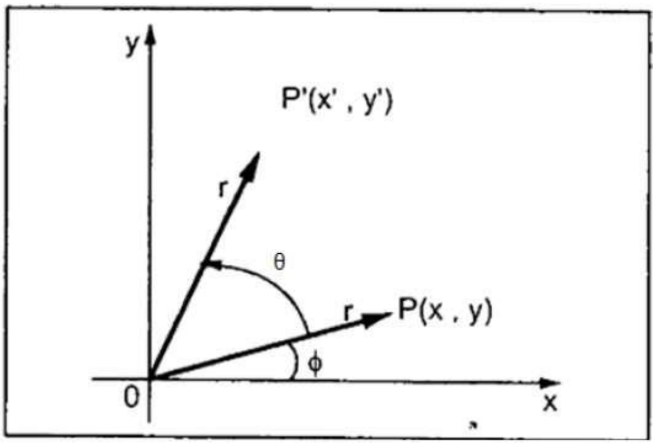
P=[X][Y]P=[X][Y] p' = [X′][Y′][X′][Y′]T = [tx][ty][tx][ty]
How we make 2D Translation matrix, have been done in class lecture. Please use class lecture method. Please here……………

We can write it as −

P’ = P + T

■ Rotation

In rotation, we rotate the object at particular angle θ thetatheta from its origin. From the following figure, we can see that the point PX,YX,Y is located at angle φ from the horizontal X coordinate with distance r from the origin.
Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P’ X′,Y′X′,Y′.



Using standard trigonometric the original coordinate of point PX,YX,Y can be represented as −

$X=r\cos\phi......(1)X=r\cos\phi......(1)$

$Y=r\sin\phi......(2)Y=r\sin\phi......(2)$

Same way we can represent the point P’ X′,Y′X′,Y′ as −

$x'=r\cos(\phi+\theta)=r\cos\phi\cos\theta-r\sin\phi\sin\theta.......(3)x'=r\cos(\phi+\theta)=r\cos\phi\cos\theta-r\sin\phi\sin\theta.......(3)$

$y'=r\sin(\phi+\theta)=r\cos\phi\sin\theta+r\sin\phi\cos\theta.......(4)y'=r\sin(\phi+\theta)=r\cos\phi\sin\theta+r\sin\phi\cos\theta.......(4)$

Substituting equation 11 & 22 in 33 & 44 respectively, we will get

$x'=x\cos\theta-y\sin\theta x'=x\cos\theta-y\sin\theta$

$y'=x\sin\theta+y\cos\theta y'=x\sin\theta+y\cos\theta$

Representing the above equation in matrix form,

$[X'Y']=[XY][\cos\theta-\sin\theta\sin\theta\cos\theta]OR[X'Y']=[XY][\cos\theta\sin\theta-\sin\theta\cos\theta]OR$

P’ = P . R

Where R is the rotation matrix

$R=[\cos\theta-\sin\theta\sin\theta\cos\theta]R=[\cos\theta\sin\theta-\sin\theta\cos\theta]$

The rotation angle can be positive and negative.

For positive rotation angle, we can use the above rotation matrix. However, for negative angle rotation, the matrix will change as shown below −

$R=[\cos(-\theta)-\sin(-\theta)\sin(-\theta)\cos(-\theta)]R=[\cos(-\theta)\sin(-\theta)-\sin(-\theta)\cos(-\theta)]$

$=[\cos\theta\sin\theta-\sin\theta\cos\theta](\because\cos(-\theta)=\cos\theta and \sin(-\theta)=-\sin\theta)=[\cos\theta-\sin\theta\sin\theta\cos\theta](\because\cos(-\theta)=\cos\theta and \sin(-\theta)=-\sin\theta)$

How we make 2D Rotation matrix, have been done in class lecture. Please use class lecture method. Please here……………

■ Scaling

To change the size of an object, scaling transformation is used. In the scaling process, you either expand or compress the dimensions of the object. Scaling can be achieved by multiplying the original coordinates of the object with the scaling factor to get the desired result.

Let us assume that the original coordinates are X,YX,Y, the scaling factors are (S_X, S_Y), and the produced coordinates are X′,Y′X′,Y′. This can be mathematically represented as shown below −

$X' = X . S_X$ and $Y' = Y . S_Y$

The scaling factor S_X, S_Y scales the object in X and Y direction respectively. The above equations can also be represented in matrix form as below −

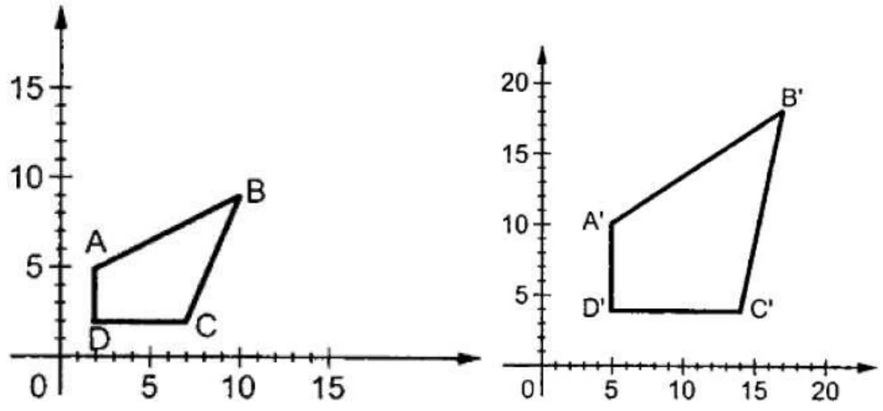
(X′Y′)=(XY)[Sx00Sy](X′Y′)=(XY)[Sx00Sy]

OR

P’ = P . S

How we make 2D Scaling matrix, have been done in class lecture. Please use class lecture method. Please here……………

Where S is the scaling matrix. The scaling process is shown in the following figure.

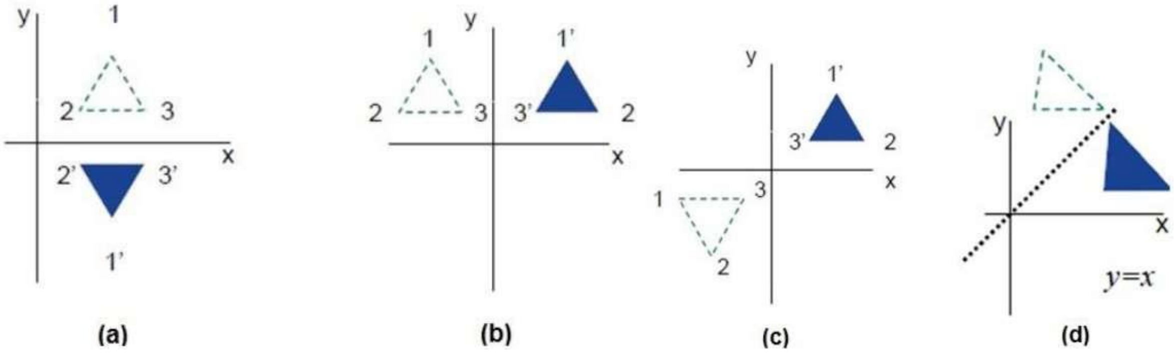


If we provide values less than 1 to the scaling factor S, then we can reduce the size of the object. If we provide values greater than 1, then we can increase the size of the object.

Reflection

Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180°. In reflection transformation, the size of the object does not change.

The following figures show reflections with respect to X and Y axes, and about the origin respectively.



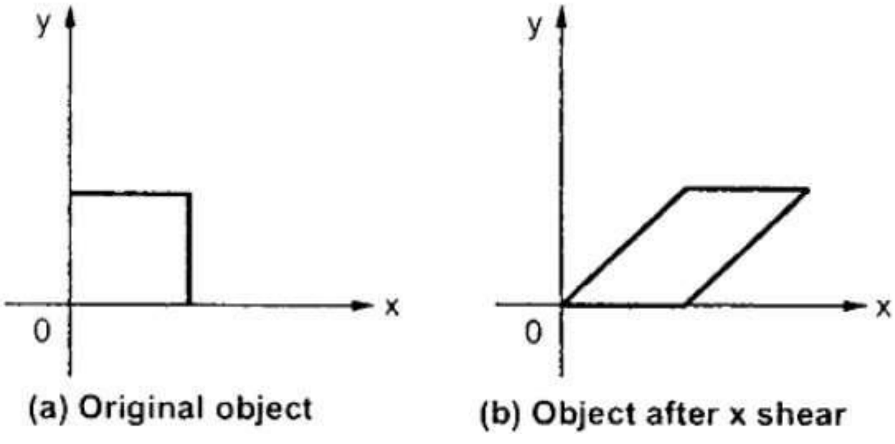
How we make 2D Mirror reflection matrix, have been done in class lecture. Please use class lecture method. Please here……………

■ Shearing

A transformation that slants the shape of an object is called the shear transformation. There are two shear transformations **X-Shear** and **Y-Shear**. One shifts X coordinates values and other shifts Y coordinate values. However; in both the cases only one coordinate changes its coordinates and other preserves its values. Shearing is also termed as **Skewing**.

X-Shear

The X-Shear preserves the Y coordinate and changes are made to X coordinates, which causes the vertical lines to tilt right or left as shown in below figure.



(a) Original object

(b) Object after x shear

The transformation matrix for X-Shear can be represented as −

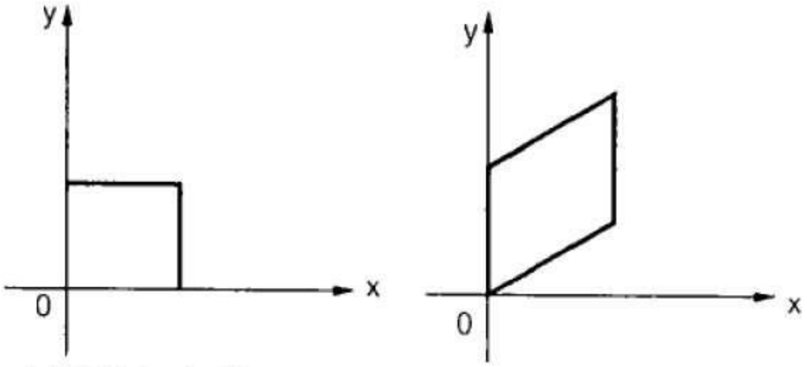
Xsh=⎡⎣⎢100shx10001⎤⎦⎥Xsh=[1shx0010001]

Y' = Y + Sh_y . X

X’ = X

Y-Shear

The Y-Shear preserves the X coordinates and changes the Y coordinates which causes the horizontal lines to transform into lines which slopes up or down as shown in the following figure.



(a) Original object (b) Object after y shear

The Y-Shear can be represented in matrix from as −

Ysh⎡⎣⎢1shy0010001⎤⎦⎥Ysh[100shy10001]

X’ = X + Sh_x . Y

Y’ = Y

How we make 2D Mirror reflection matrix, have been done in class lecture. Please use class lecture method. Please here……………

■ Composite Transformation

If a transformation of the plane T1 is followed by a second plane transformation T2, then the result itself may be represented by a single transformation T which is the composition of T1 and T2 taken in that order. This is written as T = T1·T2.

Composite transformation can be achieved by concatenation of transformation matrices to obtain a combined transformation matrix.

A combined matrix −

[T][X] = [X] [T1] [T2] [T3] [T4] …. [Tn]

Where [Ti] is any combination of

- Translation
- Scaling

- Shearing
- Rotation
- Reflection

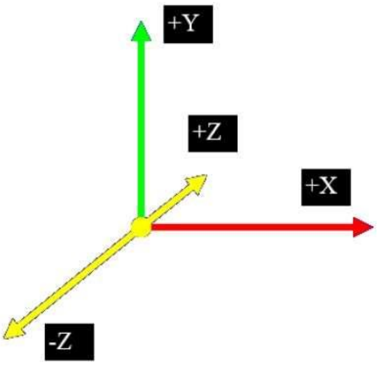
The change in the order of transformation would lead to different results, as in general matrix multiplication is not cumulative, that is [A] . [B] ≠ [B] . [A] and the order of multiplication. The basic purpose of composing transformations is to gain efficiency by applying a single composed transformation to a point, rather than applying a series of transformation, one after another.

For example, to rotate an object about an arbitrary point (X_p, Y_p), we have to carry out three steps −

- Translate point (X_p, Y_p) to the origin.
- Rotate it about the origin.
- Finally, translate the center of rotation back where it belonged.

In the 2D system, we use only two coordinates X and Y but in 3D, an extra coordinate Z is added. 3D graphics techniques and their application are fundamental to the entertainment, games, and computer-aided design industries. It is a continuing area of research in scientific visualization.

Furthermore, 3D graphics components are now a part of almost every personal computer and, although traditionally intended for graphics-intensive software such as games, they are increasingly being used by other applications.

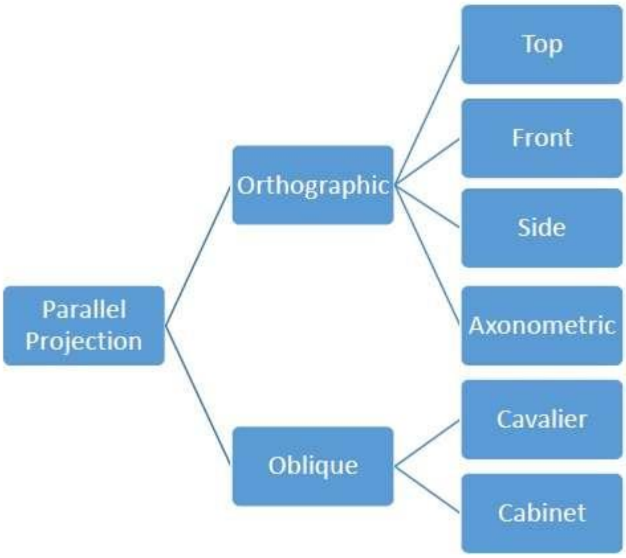


Parallel Projection

Parallel projection discards z-coordinate and parallel lines from each vertex on the object are extended until they intersect the view plane. In parallel projection, we specify a direction of projection instead of center of projection.

In parallel projection, the distance from the center of projection to project plane is infinite. In this type of projection, we connect the projected vertices by line segments which correspond to connections on the original object.

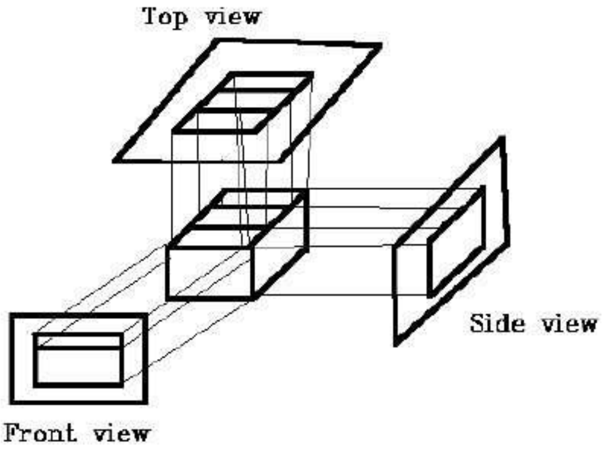
Parallel projections are less realistic, but they are good for exact measurements. In this type of projections, parallel lines remain parallel and angles are not preserved. Various types of parallel projections are shown in the following hierarchy.



Orthographic Projection

In orthographic projection the direction of projection is normal to the projection of the plane. There are three types of orthographic projections −

- Front Projection
- Top Projection
- Side Projection

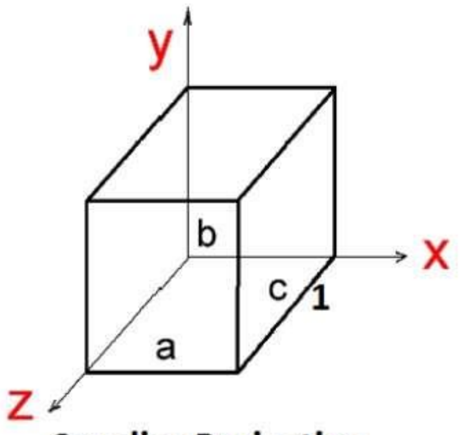


■ Oblique Projection

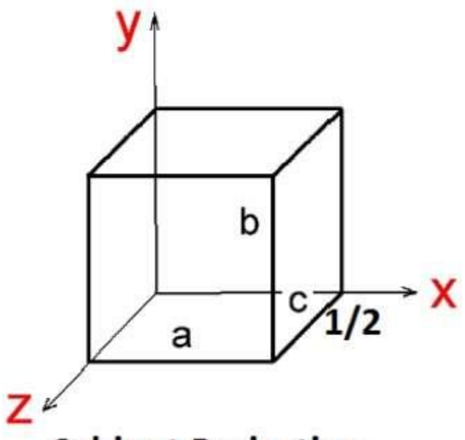
In oblique projection, the direction of projection is not normal to the projection of plane. In oblique projection, we can view the object better than orthographic projection.

There are two types of oblique projections − **Cavalier** and **Cabinet**. The Cavalier projection makes 45° angle with the projection plane. The projection of a line perpendicular to the view plane has the same length as the line itself in Cavalier projection. In a cavalier projection, the foreshortening factors for all three principal directions are equal.

The Cabinet projection makes 63.4° angle with the projection plane. In Cabinet projection, lines perpendicular to the viewing surface are projected at ½ their actual length. Both the projections are shown in the following figure –



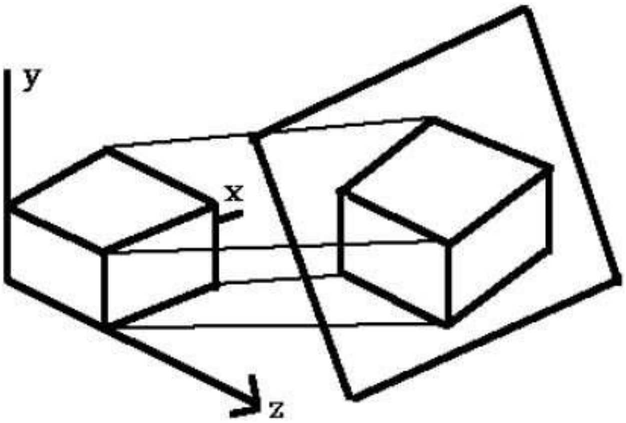
Cavalier Projection



Cabinet Projection

■ Isometric Projections

Orthographic projections that show more than one side of an object are called **axonometric orthographic projections**. The most common axonometric projection is an **isometric projection** where the projection plane intersects each coordinate axis in the model coordinate system at an equal distance. In this projection parallelism of lines are preserved but angles are not preserved. The following figure shows isometric projection −

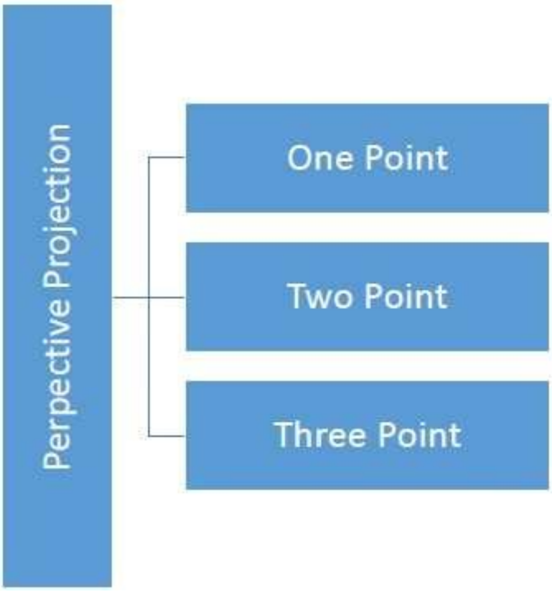


Perspective Projection

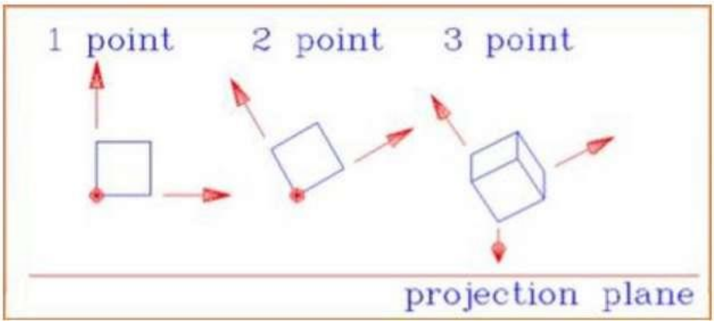
In perspective projection, the distance from the center of projection to project plane is finite and the size of the object varies inversely with distance which looks more realistic.

The distance and angles are not preserved and parallel lines do not remain parallel. Instead, they all converge at a single point called **center of projection** or **projection reference point**. There are 3 types of perspective projections which are shown in the following chart.

- **One point** perspective projection is simple to draw.
- **Two point** perspective projection gives better impression of depth.
- **Three point** perspective projection is most difficult to draw.



The following figure shows all the three types of perspective projection −



Composite Transformation:

A number of transformations or sequence of transformations can be combined into single one called as composition. The resulting matrix is called as composite matrix. The process of combining is called as concatenation.

Suppose we want to perform rotation about an arbitrary point, then we can perform it by the sequence of three transformations

1. Translation
2. Rotation
3. Reverse Translation

The ordering sequence of these numbers of transformations must not be changed. If a matrix is represented in column form, then the composite transformation is performed by multiplying matrix in order from right to left side. The output obtained from the previous matrix is multiplied with the new coming matrix.

Example showing composite transformations:

The enlargement is with respect to center. For this following sequence of transformations will be performed and all will be combined to a single one

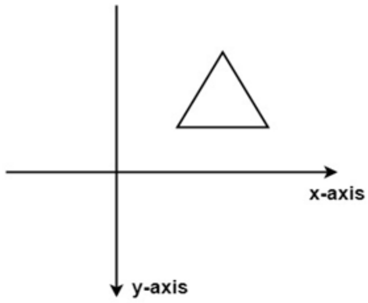
Step1: The object is kept at its position as in fig (a)

Step2: The object is translated so that its center coincides with the origin as in fig (b)

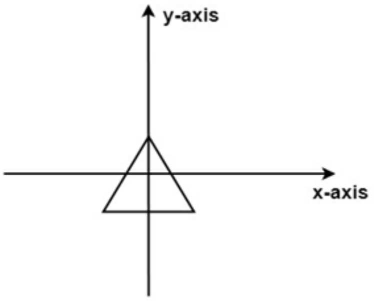
Step3: Scaling of an object by keeping the object at origin is done in fig (c)

Step4: Again translation is done. This second translation is called a reverse translation. It will position the object at the origin location.

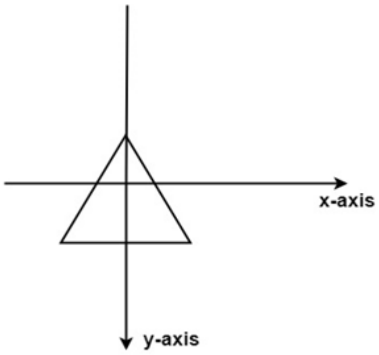
Above transformation can be represented as $T_V.ST_V^{-1}$



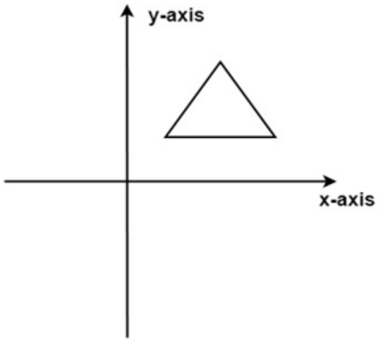
(original position of object)
Fig:(a)



(object is translated to origin)
Fig:(b)



(object is enlarged)
Fig:(c)



(object is retranslated to original position)
Fig:(d)

Note: Two types of rotations are used for representing matrices one is column method. Another is the row method.

Row Method

1. Translation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

2. Scaling

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ S_x & S_y & 1 \end{bmatrix}$$

Column Method

1.Translation

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

2. Scaling

$$\begin{bmatrix} 1 & 0 & S_x \\ 0 & 1 & S_y \\ 0 & 0 & 1 \end{bmatrix}$$

Advantage of composition or concatenation of matrix:

1. It transformations become compact.
2. The number of operations will be reduced.

3. Rules used for defining transformation in form of equations are complex as compared to matrix.

Composition of two translations:

Let t_1 t_2 t_3 $t_4$are translation vectors. They are two translations P_1 and P_2. The matrix of P_1 and P_2 given below. The P_1 and $P_2$are represented using Homogeneous matrices and P will be the final transformation matrix obtained after multiplication.

$$P_1 = \begin{bmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \\ 0 & 0 & 1 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 0 & t_3 \\ 0 & 1 & t_4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & t_1 + t_2 \\ 0 & 1 & t_3 + t_4 \\ 0 & 0 & 1 \end{bmatrix}$$

Above resultant matrix show that two successive translations are additive.

Composition of two Rotations: Two Rotations are also additive

Composition of two Scaling: The composition of two scaling is multiplicative. Let S_{11} and $S_{12}$are matrix to be multiplied.

$$S_{11} = \begin{bmatrix} S_1 & 0 & 0 \\ 0 & S_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{12} = \begin{bmatrix} S_3 & 0 & 0 \\ 0 & S_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = S_{11} * S_{12} = S_{11}(S_1.S_2).S_{12}(S_3.S_4) = S(S_1.S_2.S_3.S_4)$$

Inverse Transformations

These are also called as opposite transformations. If T is a translation matrix than inverse translation is representing using T^{-1}. The inverse matrix is achieved using the opposite sign.

Example1: Translation and its inverse matrix

Translation matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -T_x & -T_y & -T_z & 1 \end{pmatrix}$$

Example2: Rotation and its inverse matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} -\cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & -\cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

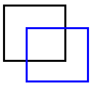
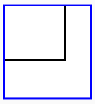
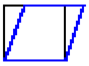
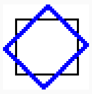
■ AFFINE TRANSFORMATION

Linear mapping method using affine transformation

Affine transformation is a linear mapping method that preserves points, straight lines, and planes. Sets of parallel lines remain parallel after an affine transformation.

The affine transformation technique is typically used to correct for geometric distortions or deformations that occur with non-ideal camera angles. For example, satellite imagery uses affine transformations to correct for wide angle lens distortion, panorama stitching, and image registration. Transforming and fusing the images to a large, flat coordinate system is desirable to eliminate distortion. This enables easier interactions and calculations that don’t require accounting for image distortion.

The following table illustrates the different affine transformations: translation, scale, shear, and rotation.

Affine Transform	Example
Translation	
Scale	
Shear	
Rotation	

What are Pointing Device and Positioning Devices and its Techniques?

Pointing technique refers to look at the items already on the screen whereas the positioning technique refers to position the item on the screen to a new position, i.e., the old current position. The user indicates a position on the screen with an input device, and this position is used to insert a symbol.

The pointing and positioning are interactions used in computer graphics. A pointing device is also called a **Pointing tool.** A pointing tool is a hardware component, used to shift the cursor in a computer program or a graphical user interface (GUI) operating system. The standard features of the mouse are click, point, drag, and drop.
According to Fitts's theorem, we can use the pointing devices to estimate the speed by which we can point at a higher speed. **The positioning technique** is used to move the object from one position to another position.
There are following pointing and positioning devices given below:

- Mouse
- Light Pen
- Joystick
- Graphics Tablet
- Pointing Stick

Mouse

A mouse is a lightweight, small device used to control the movement of the cursor on a computer screen. Usually, the mouse moves on a flat surface. A mouse contains two or three buttons. Every button performs a different task. The mouse can be considered as a positioning and pointing device.

Functions of Mouse

There are following functions of a mouse:

- Clicking
- Right-clicking
- Dragging
- Scrolling
- Double Clicking

Light Pen

A light pen is a handheld, light-sensitive device. It is like a pen. The light pen connects to the computer by a wire. A light pen contains a device at the tip that emits light. Mostly, light pens are used by graphics designers, engineers, and illustrators. The light pen sends the information to the computer when we touch the pen to the computer screen.

Joystick

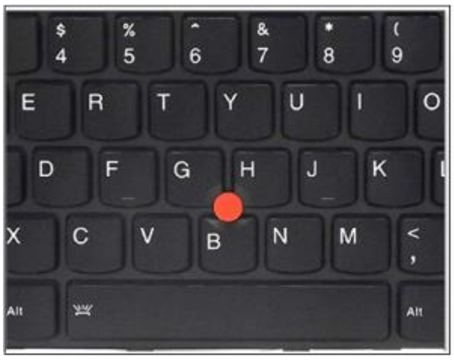
A joystick has two parts a base and a stick. We can move the stick in every direction to shift the object from one position to another position on the computer screen. The joystick is similar to a mouse or trackball. Mostly the joystick is used to play computer games. A joystick has a fast interface and also used in computer-aided design.

Graphics Tablet

A graphics tablet is also called a **digitizer**. A graphics tablet contains a flat pad on which we can draw an image or shape with a pen called **stylus.** The image gets drawn on the screen as we draw on the flat pad. It is mostly used by graphics designers to produce accurate drawings and pictures.

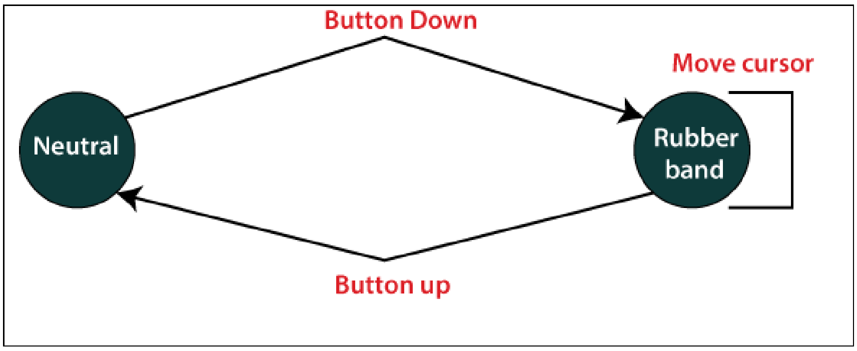
Pointing Stick

The pointing device is also known as the Pressure-Sensitive device. It exists between the keys of the keyboard. It is similar to a pencil eraser. The pointing stick works like a mouse. When we touch the finger on the pointing stick, the cursor moves on the computer screen. A pointing stick does not need any extra space like a mouse. It is mostly used in notebooks.



Rubber band or Elastic Technique

The rubber band technique was introduced by **Blaisdell** in **1958** and refined by **Barron** in **1963.** It is a positioning method. The rubber band is also named as an elastic band, gum band, or lacky band. It is a loop that is usually used to hold multiple objects together. It is a useful and simple technique for positioning the objects. We can easily draw geometric shapes, i.e., lines, ellipse, circle, and rectangle, etc. on the computer screen by rubber band technique. In this technique, we can perform positioning on the line by positioning the two endpoints. In this, we shift the first point to the last point; the computer draws a line from the first point to cursor point. The technique called rubber band or elastic band because the line stretched among with first point and the cursor.



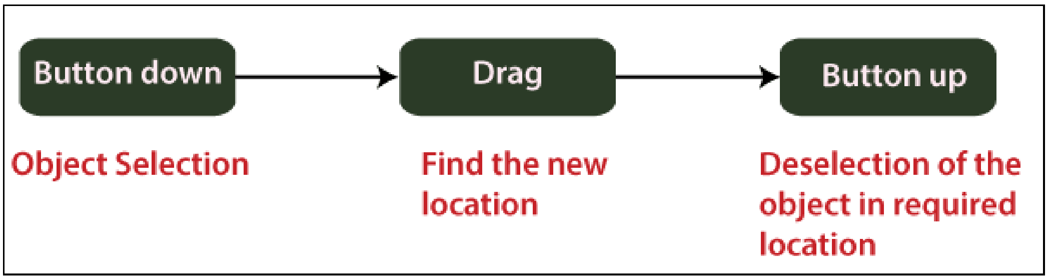
Advantages of Rubber Band Technique

- It is simple and easy to implement the technique.
- We can draw many geometric shapes by using this technique.

Disadvantages of Rubber Band Technique

- It is quite expensive.
- The rubber band technique needs good software and a fast processor.

Dragging



Dragging is a technique used in interactive image drawing. In this process, we can change the position of an object from one to another place. We can drag the shapes or objects with the cursor.

Steps to drag an object

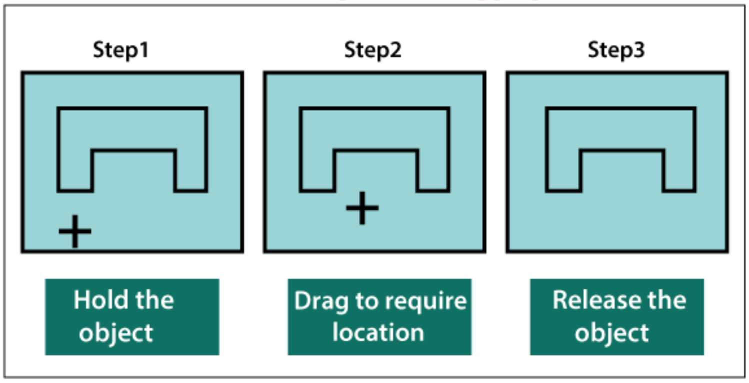
Step 1: First, we choose an object.

Step 2: Now, we shift the cursor in the direction where we want to shift the object.

Step 3: Select the object that follows the cursor path.

Dragging the object to different positions is very useful in applications where we want to show the various possible location before choosing the final location.

Basic Diagram of Dragging


Step1
Step2
Step3
Hold the object
Drag to require location
Release the object