UNIT-4

Subject - Ordinary Differential equations
Class - B.Sc. | B.A. (2nd Sem)

Chapter-7 (Ordinary Simultaneous Differential equations)

In this chapter we will discuss differential equations in which there is only one independent variable and two or more than two dependent variables.

We need as many simultaneous equations as the number of dependent variables.

eg. If we have two dependent variables then there will be two differential equations simultaneously.

Let t be independent variable and x, y be two independent variables.

<u>First</u> <u>method</u> :- Use of operator D.

SUNDAY 01

① Put $D = \frac{d}{dt}$

② Now try to eliminate the variable y (you can also remove x).

③ ~~pb~~ After eliminating y, we obtain a differential equation

Su	1	8	15	22	29
M	2	9	16	23	30
T	3	10	17	24	31
W	4	11	18	25	
T	5	12	19	26	
F	6	13	20	27	
S	7	14	21	28	

DECEMBER 2013

in x and t which when solved gives value of x in terms of t.

(4) Put this value of x back to get differential equation in terms of y and t.

(5) Then solve this, we get value of y in interms of t.

Example

eg1 Solve the simultaneous equations

$$\frac{dx}{dt} + 5x + y = e^{t} \rightarrow (1)$$

$$\frac{dy}{dt} - x + 3y = e^{2t} \rightarrow (2)$$

Solution :- step 1 :- Put $\frac{d}{dt} = D$ in eq^{ns} (1) & (2)

$$Dx + 5x + y = e^{t}$$

$$Dy - x + 3y = e^{2t}$$

we get

$$(D+5)x + y = e^{t} \rightarrow (3)$$

$$-x + (D+3)y = e^{2t} \rightarrow (4)$$

Now we will eliminate y between (3) and (4) [Note:- you can also eliminate x, that depends on you.]

We will make coefficient of y same in both the equations.

Multiply eq^n ③ by $(D+3)$, we get

$$(D+5)(D+3)x + (D+3)y = (D+3)e^t \rightarrow ⑤$$

Now we get equations ⑤ and ④

$$(D+5)(D+3)x + (D+3)y = (D+3)e^t$$
$$-x + (D+3)y = e^{2t}$$
$$+ \quad - \quad -$$

By subtracting these two

$$(D+5)(D+3)x + x = (D+3)e^t - e^{2t}$$
$$(D^2+8D+16)x = De^t + 3e^t - e^{2t}$$

$$(D^2+8D+16)x = e^t + 3e^t - e^{2t} \quad \left[\because De^t = \frac{d}{dt}(e^t) = e^t\right]$$

$$(D^2+8D+16)x = 4e^t - e^{2t} \rightarrow ⑥$$

Now solve diff. eq^n ⑥ to get x in terms of t.
As studied earlier in chapter 4.

Solving eq^n ⑥ :-

Aux. Equation $D^2+8D+16=0$

$$D = -4, -4$$

$$C.F. = (c_1 + c_2 t)e^{-4t}$$

$$P.I. \quad \frac{1}{(D+4)^2}(4e^t - e^{2t})$$

$$4\frac{1}{(D+4)^2}e^t - \frac{1}{(D+4)^2}e^{2t}$$

put $D=1$ put $D=2$

$$4 \cdot \frac{e^{t}}{25} - \frac{e^{2t}}{36}$$

Hence by solving eq^n (6)

$$x = \underbrace{(C_1 + C_2 t)e^{-4t}}_{C.F.} + \underbrace{\frac{4e^{t}}{25} - \frac{e^{2t}}{36}}_{P.I.} \rightarrow (7)$$

Differentiate eq^n (7) w.r.t t

$$\frac{dx}{dt} = -4(C_1 + C_2 t)e^{-4t} + C_2 e^{-4t} + \frac{4e^{t}}{25} - \frac{e^{2t}}{18} \rightarrow (8)$$

Last step :- we need y in terms of t

Now from eq^n (1), value of y is

$$y = e^{t} - \frac{dx}{dt} - 5x$$

Put value of x and $\frac{dx}{dt}$ from (7) & (8)

$$y = e^{t} + \underbrace{4(C_1 + C_2 t)e^{-4t} - C_2 e^{-4t} - \frac{4}{25}e^{t} + \frac{e^{2t}}{18}}_{-\frac{dx}{dt}}$$

$$-5\Big[\underbrace{(C_1 + C_2 t)e^{-4t} + \frac{4}{25}e^{t} - \frac{e^{2t}}{36}}_{x}\Big]$$

$$= e^{t} + 4(C_1 + C_2 t)e^{-4t} - C_2 e^{-4t} - \frac{4}{25}e^{t} + \frac{e^{2t}}{18}$$

$$- 5(C_1 + C_2 t)e^{-4t} - \frac{4}{5}e^{t} + \frac{5}{36}e^{2t}$$

We get by solving,

$$y = -(C_1+2C_2 t)e^{-4t} + \frac{1}{25}e^{t} + \frac{7}{36}e^{2t}$$

Hence, required solution is

$$x = (C_1 + C_2 t)\, e^{-4t} + \frac{4}{25}e^{t} - \frac{e^{2t}}{36}$$

$$y = -(C_1 + 2C_2 t)\, e^{-4t} + \frac{1}{25}e^{t} + \frac{7}{36}e^{2t}$$