BSc. II year Sub :→ Special functions and Integral Transforms

Chapter - 1 (Power Series)

1. Solve $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$ in series

2. Solve the following differential equation in power series:- $2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-1)y = 0$

3. Solve $x\frac{dy}{dx} - y - x - 1 = 0$ in powers of $(x-1)$

4. Determine the radius of convergence of the power series :- $\sum_{m=0}^{\infty} \frac{(x-2)^{2m}}{m!}$

5. Obtain the power series solution of. $\frac{d^2y}{dx^2} + (x-3)\cdot\frac{dy}{dx} + y = 0$ in powers of $(x-2)$

6. Find the series solution of the following differential equation about 0:- $x\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + xy = 0$

7. Determine the radius of convergence of the power series $\sum_{m=0}^{\infty}\left(\frac{2}{3}\right)^m x^{2m}$

8. Solve the following differential equations in power series:- $9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0$

9. Solve:→ $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ in series about $x=0$

10. Find the power series solution of the following differential in powers of x
$(2+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (1+x)y = 0$

11 Solve the following differential equation in power series :- $3x\frac{d^2y}{dx^2} - (x-2)\frac{dy}{dx} - 2y = 0$

12 Show that $x=1$ is a regular solution point of

$$(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$$

13 Write solution of $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

14 Find the power-series solution of the differential Equation

$$(x^2-4x+5)\frac{d^2y}{dx^2} + (x-2)\frac{dy}{dx} - (x-2)y = 0$$ in powers of $(x-2)$

15 Define Radius of Convergence of a power series in terms of Coefficients of series.

16 Find the radius of convergence of power series

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{5^m}(x+1)^{3m}$$

17 Find radius of convergence of power series $\sum_{m=0}^{\infty}(m+1)!(x-a)$

18 Define Analytic function and write its two examples?

19 Solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ in series

20 Solve $x(1-x)\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

21 Solve the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ in series.

22 Find the solution of the following equation

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0$$

23 Find the power series solution of the Initial value problem

$(x^2-1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0$, where $y(2)=4$, $y'(2)=6$.

24. Find the series solution of the differential equation about $x=0$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2-4)y=0$$

25. Find the series solution of following differential equation → $(x^3-1)\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+xy=0$

26. Solve → $x^2\frac{d^2y}{dx^2}+(x+x^2)\frac{dy}{dx}+(x-9)y=0$ in series.

27. Find the power series solution of

$$(1-x^2)\frac{d^2y}{dx^2}+2y=0,\quad \text{where } y(0)=4 \text{ and } y'(0)=5.$$

28. Find the series solution of the differential Equation

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+x^2y=0, \text{ about } 0.$$

29. Solve the following differential equation about $x=0$

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-\left(x^2+\frac{5}{4}\right)y=0$$

30. Solve $y''-xy'=e^{-x}$, where $y(0)=2$ and $y'(0)=-3$, where y' and y'' have usual meanings.

31. Find the power series solution in powers of $(x-1)$ of the initial value problem

$$x\frac{d^2y}{dx^2}+\frac{dy}{dx}+2y=0,\quad y(1)=1,\ \frac{dy}{dx}(\text{at } 1)=2.$$

32. Solve $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = 0$ in series.

33. Find the power series solution about $x=0$

$$(x^2-1)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$$

34. Show that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$.

35. Find the orthogonal relation of Bessel's function $J_n($

36. Show that $J_{-n}(x) = (-1)^n J_n(x)$, where n is any integ

37. Find the solution of $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{1}{4}y = 0$ in terms of Bessel's function.

Chapter-2 (Bessel's Equation and Function)

1. State and Prove the orthogonality relation of Bessel's function.

2. Solve the Bessel's equation of order n when n is real constant.

3. Define generating function of Bessel's functions and hence find the integral form of the Bessel's functions $J_n(x)$ for all integral values of n.

4. Show that $\frac{\pi x}{2}\left[J_{1/2}^2(x) + J_{-1/2}^2(x)\right] = 1$

5. Show that :- $J_n'(x) = \frac{1}{2}\left[J_{n-1}(x) - J_{n+1}(x)\right]$

6. Show that under the transformation $y = \frac{u}{\sqrt{x}}$ the Bessel's equation becomes $\frac{d^2y}{dx^2} + \left[1 + \frac{1-4n^2}{4x^2}\right]u = 0$. Hence, find the solution of this equation.

7. Show that $J_{-1/2}(x) = J_{1/2}(x)\cot x$

8. Find the solution of the following equation in term of Bessel's function
$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \frac{1}{2}xy = 0$$

9. Show that $J_4(x) = \left[\frac{48}{x^3} - \frac{8}{x}\right]J_1(x) + \left[1 - \frac{24}{x^2}\right]J_0(x)$

10. Prove that :-
$$\int J_3(x)\,dx = C - J_2(x) - \frac{2}{x}J_1(x)$$

11. Prove that $J_n'(x) - \frac{n}{x}J_n(x) = -J_{n+1}(x)$

12. Find the solution of $\frac{d^2y}{dx^2} + \left(9x - \frac{20}{x^2}\right)y = 0$ in terms of Bessel's function.

13. Show that $\frac{d}{dx}\left[xJ_n(x)J_{n+1}(x)\right] = x\left[J_n^2(x) - J_{n+1}^2(x)\right]$

14 Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\, d\theta$ for all integral values of n.

15 Find the solution of $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{1}{4}y = 0$ in terms of Bessel's function.

16 Prove that → $J_n(x) = \frac{1}{\pi}\int_0^{\pi} \cos(n\theta - x\sin\theta)\, d\theta$ for all integral values of n.

17 Prove that → $e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x)\, t^n$

18 Find ~~the solution of differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{1}{4}y = 0$ in terms of Bessel's function~~

19 Prove that → $\cos(x\sin\theta) = J_0(x) + 2[J_2(x)\cos 2\theta + J_4(x)\cos 4\theta + \cdots]$

20 Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

21 Write the Bessel function $J_0(x)$ in the form of series

22 Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$ with usual notation.

23 Solve in terms of Bessel's function –

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{1}{9x^2}\right)y = 0$$

24 Write the Bessel function $J_1(x)$ in the form of series.

25 Prove that –

$\sin(x\sin\theta) = 2[J_1(x).\sin\theta + J_3(x).\sin 3\theta + J_5(x)\sin 5\theta + \cdots]$

26 Show that $x^n J_n(x)$ is the solution of $x\frac{d^2y}{dx^2} + (1-2n)\frac{dy}{dx} + xy = 0$

Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3-x^2}{x^2}\sin x - \frac{3}{x}\cos x\right]$

28 Show that $J_{-n}(x) = (-1)^n J_n(x)$, where n is any integer

29 Show that $\frac{d}{dx}\left[J_n^2(x)\right] = \frac{x}{2n}\left[J_{n-1}^2(x) - J_{n+1}^2(x)\right]$

30 Show that $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$

31 Show that $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$

32. Find the orthogonal relation of Bessel's function $J_n(x)$

33. Show that $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$

Chapter - 3 Legendre's Equation

(1) To prove that $P_n(x) = \frac{1}{n!\,2^n} \frac{d^n}{dx^n} (x^2-1)^n$

(2) Derive derivation of Legendre's polynomials from Rodrigue's Formula

(3) State & Prove generating function for $P_n(x)$

(4) Express $4x^3-2x^2-3x+8$ in term of Legendre's polynomials.

(5) Show that $P_n(-x) = (-1)^n P_n(x)$

(6) Prove that $\frac{1-t^2}{(1-2x+t^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1)\, t^n P_n(x)$

(7) Show that $\int_{-1}^{1} P_n(x)\,dx = \begin{cases} 2, & \text{if } n=0 \\ 0, & \text{if } n \geq 1 \end{cases}$

(8) Solve the Legendre equation when the parameter is equal to zero

(9) Verify that Legendre polynomial $P_3(x) = \frac{1}{2}(5x^3-3x)$ satisfies the Legendre equation when the parameter n is equal to 3.

(10) Express $x^4+2x^3+2x^2-x-3$ in term of Legendre polynomial

(11) Show that $(n+1)P_{n+1}(x) + n P_{n-1}(x) = (2n+1)\,x\,P_n(x)$

(12) State & Prove Orthogonality of Legendre polynomial.

(13) Prove that $\int_{-1}^{1} x\, P_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2-1}$

(14) Prove that

$$\int^{1} x^2 P_{n+1}(x)\, P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

(15) Prove that $\int_{-1}^{1} P_n(x)\,(1-2xt+t^2)^{-1/2}\,dx = \frac{2t^n}{2n+1}$

(16) Prove that

$$\int_{-1}^{1}(1-x^2)\,P_m'(x)\,P_n'(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2m(m+1)}{2m+1} & \text{if } m = n \end{cases}$$

(17) Prove that $\int_{x}^{1} P_n(x)\,dx = \frac{1}{2n+1}\left[P_{n-1}(x) - P_{n+1}(x)\right]$

(18) Show that $P_6(x) = \frac{1}{16}\left[231x^6 - 315x^4 + 105x^2 - 5\right]$

(19) Show that $\frac{1}{5}\left[3P_1(x) + 2P_3(x)\right] = x^3$

(20) Using Rodrigue's formula, show that $P_n(x)$ satisfies the differential equation

$$\frac{d}{dx}\left[(1-x^2)\frac{d}{dx}P_n(x)\right] + n(n+1)P_n(x) = 0$$

(21) Prove that $\int_{-1}^{1} P_m(x)\,P_n(x)\,dx = 0$, when $m \neq n$

and $\int_{-1}^{1}\left[P_m(x)\right]^2 dx = \frac{2}{2m+1}$, when $m = n$

(22) Prove that $\int_{-1}^{1} P_0(x)\,dx = 2$

(23) Using Rodrigue's formula find $P_1(x)$ and $P_2(x)$

(24) Write Legendre's equation and what do you mean by Legendre Polynomial?

25. Show that $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$ $|x| \leq 1, |t| < 1$ where $P_n(x)$ is Legendre's func of order n.

26 Show that $(1-x^2)\,P_n'(x) = (n+1)\left[x P_n(x) - P_{n+1}(x)\right]$.

Chapter-4 Hermite's Equation

(1) Derive Hermite's Polynomial for some value of n.

(2) Evaluates the values of $H_{2n}(0)$ and $H_{2n+1}(0)$

(3) State & Prove Rodrigue's formula for $H_n(x)$

(4) To show that $H_n(x) = 2^n \left[\exp\left(-\frac{1}{4} \frac{d^2}{dx^2} \right) \right] x^n$

(5) To show that $H_n'(x) = 2n H_{n-1}(x), \; n \geq 1$

(6) Show that $2x H_n(x) = 2n H_{n-1}(x) + H_{n+1}(x), \; n \geq 1$

(7) Show that $H_n'(x) = 2x H_n(x) - H_{n+1}(x)$

(8) Prove that if $m<n$, $\frac{d^m}{dx^m}[H_n(x)] = \frac{2^m (n)!}{(n-m)!} H_{n-m}(x)$

(9) Show that $H_1(x) = 2x H_0(x)$

(10) Show that $P_n(x) = \frac{2}{\sqrt{\pi}\, n!} \int_0^\infty t^n e^{-t^2} H_n(xt)\, dt$.

(11) Using Rodrigue's formula for $H_n(x)$ and integrating by parts respectively, show that

$$\phi = \int_{-\infty}^{\infty} \exp(-x^2) H_n(x) H_m(x)\, dx \begin{cases} 0 & \text{if } m \neq n \\ 2^n n! \sqrt{\pi} & \text{if } m = n \end{cases}$$

(12) Express $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of the Hermitian Polynomial

(13) Expand e^{2x} in a series of Hermite's polynomials.

(14) Prove that $H_5(x) = 32x^5 - 160x^3 + 120x$

(15) Prove that $\int^{\infty} x^2 e^{-x^2} [H_n(x)]^2 dx = \sqrt{\pi}\, 2^n n! \left(n + \frac{1}{2}\right)$

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(16) If $\phi_n(x) = e^{-\frac{x^2}{2}} H_n(x)$, where $H_n(x)$ is a Hermite's polynomial then

$$\int_{-\infty}^{\infty} \phi_m(x)\,\phi_n'(x)\,dx = \begin{cases} 0 & \text{if } m \neq n \pm 1 \\ 2^{n-1}\, n!\, \sqrt{\pi} & \text{if } m = n-1 \\ -2^{n}(n+1)!\,\sqrt{\pi} & \text{if } m = n+1 \end{cases}$$

(17) Evaluate the value of $H_0(x), H_1(x), H_2(x), H_3(x)$

(18) Show that $H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120$

(19) Show that

$$\sum_{k=0}^{n} \frac{H_k(x)\,H_k(y)}{2^k (k!)} = \frac{H_{n+1}(y)\,H_n(x) - H_{n+1}(x)\,H_n(y)}{2^{n+1}(n!)\,(y-x)}$$

(20) Show that $H_n''(x) = 4n(n-1)\,H_{n-2}(x)$

(21) Show that → $e^{2tx - t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$

Chapter-5 (Laplace Transforms)

1. Find the Laplace transform of the function $\sinh^3 2t$.

2. Find the Laplace transform of $e^{-at}\sinh bt$.

3. If the Laplace transform of the function $f(t)$ for $t \geq 0$ is $F(s)$, then show that

$$L[(\cosh at) f(t)] = \frac{1}{2}[F(s-a) + F(s+a)]$$

4. Find the Laplace transform of $\sinh 3t \cos^2 t$.

5. Find the Laplace transform of the function
$e^{-2t}\sin t \cos 3t$.

6. Show that $L\left(\sinh \frac{1}{2}t \sin \frac{1}{2}\sqrt{3}t\right) = \frac{\sqrt{3}\, s}{2(s^4+s^2+1)}$

7. Find the Laplace transform → $e^{2t}\sin t \cos^3 t$

8. State and Prove Second Shifting Theorem

9. Find the Laplace transform of
$f(t) = |t-1| + |t+1|,\ t \geq 0$

10. Find the Laplace transform of

$$g(t) = \begin{cases} \cos\left(t - \frac{2\pi}{3}\right), & t > \frac{2\pi}{3} \\ 0, & 0 < t < \frac{2\pi}{3} \end{cases}$$

11. Find the Laplace transform of

$$f(x) = \begin{cases} 0, & 0 < t < 1 \\ t, & 1 < t < 2 \\ 0, & t > 2 \end{cases}$$

ind the Laplace transform of the function

$$f(t) = \begin{cases} t, & 0<t<2 \\ 3, & t>2 \end{cases}$$

13 Find the Laplace transform of the function

$$f(t) = \begin{cases} \frac{t}{T}, & 0<t<T \\ 1, & t>T \end{cases}$$

14 Find the Laplace transform of the function

$$f(t) = \begin{cases} \frac{1}{\varepsilon} & \text{if } 0 \le t \le \varepsilon \\ 0 & \text{if } t > \varepsilon \end{cases}$$

15 Find → $g(t) = \begin{cases} \sin\left(t-\frac{\pi}{6}\right), & t>\frac{\pi}{6} \\ 0, & 0<t<\frac{\pi}{6} \end{cases}$

16 Find the Laplace transform of $\sin at - at\cos at$

17 Find the Laplace transform of $t^2 \cos at$

18 Evaluate $L(t\,e^{-t}\sin 3t)$

19 Evaluate $L\left(\frac{e^{-t}\sin t}{t}\right)$

20 Evaluate $L\left(\frac{1-\cos 2t}{t}\right)$

21 State & prove Laplace Transform of a Periodic function.

22 Find the Laplace Transform of periodic function

$$f(t) = \frac{kt}{T},\ 0<t<T\ ;\ f(t+T) = f(t)$$

23 Find the Laplace transform of function

$$(t^2-3t+2)\sin 3t$$

24 Find the Laplace transform of function

$$t^2 \sin at$$

Given $L\left[2\sqrt{\frac{t}{\pi}}\right] = \frac{1}{s^{3/2}}$, Show that $L\left(\frac{1}{\sqrt{\pi t}}\right) = \frac{1}{s^{1/2}}$

26 State & Prove Laplace Transform of integrals.

27 Evaluate $\int_0^\infty t e^{-2t} \cos t \, dt$

28 Evaluate $\int_0^\infty t e^{-t} \sin^4 t \, dt$

29 Show that $\int_0^\infty \sin x^2 \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$

30 Evaluate $\int_0^\infty t e^{-2t} \sin t \, dt$

31 Show that $\int_0^\infty \cos x^2 \, dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$

32 Find $L J_0(t)$

33 Find $L J_0(at)$

34 To prove that $\int_0^\infty J_0(t) \, dt = 1$.

35 Find Laplace transform of exponential integral function $\int_t^\infty \frac{e^{-x}}{x} dx$, which is denoted by $E(t)$.

36 Find the Laplace transform of sine integral ie $\int_0^t \frac{\sin u}{u} du$.

37 Find the Laplace transform of the function $e^{-2t} \sin 4t$.

38 ~~Find~~ $\cancel{L\left[\frac{1}{t}\sin t\right]}$ Find the Laplace transform of the function $e^{4t} \sin 2t \cos t$

39 Find the Laplace transform of the function $f(t) = \begin{cases} \cos t, & 0 < t < 2\pi \\ 0, & t > 2\pi \end{cases}$

40. Find the Laplace transform of the function —

$$\left[\sqrt{t}-\frac{1}{\sqrt{t}}\right]^3$$

41. Find the Laplace transform of $\sin 6t \sin 4t$.

42. Find Laplace transform of $\frac{\sin^2 t}{t}$

~~43. Find the Laplace transform of exponential integral function $E(t) = \int_t^\infty \frac{e^{-x}}{x}\,dx$.~~

44. Find $L[\sin t/2 \sin 3t/2]$

45. Evaluate → $L[e^{2t} + 4t^3 - 2\sin 3t + 3\cos 3t]$

46. Find laplace transform of $t\sin^2 t$.

47. Find the Laplace transforms of $e^{-2t}\sin t \cos 3t$.

Chapter-7

Use of Laplace transforms in Integral Equations

① Solve the Integral equation $f(t) = 1 + \int_0^t f(u).\sin(t-u)\,du$ and verify your solution.

② Solve the integral equation $f(t) = 1 + 2\int_0^t f(t-u)e^{-2u}\,du$.

③ Convert the differential equation $f''(t) - 3f'(t) + 2f(t) = 4\sin t$ into an integral equation, where $f(0) = 1$, $f'(0) = -2$

④ Solve the following integro-differential equation by using Laplace transforms

$$f'(t) + 3f(t) + 2\int_0^t f(u)\,du = t,\ f(0) = 1, \text{ given } y(0) = 1$$

⑤ Solve the integral equation $\int_0^t \frac{f(u)}{(t-u)^{1/3}}\,du = t(1+t)$

⑥ Solve $f'(t) = t + \int_0^t f(t-u)\cos u\,du$, given $f(0) = 4$

⑦

11 Solve $(D^2+2)x - Dy = 1$

$Dx + (D^2+2)y = 0$

where $x(0) = 0$, $x'(0) = 0$, $y(0) = 0$, $y'(0) = 0$ by Laplace method.

Chapter - 8 (Solution of Differential Equation by Laplace Transforms) :

① Solve the following equation by transform method

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}, \text{ where } y(0) = y'(0) = 1$$

② Solve $\frac{d^4y}{dt^4} + 2\frac{d^2y}{dt^2} + y = \sin t$ by transform method,

where $y(0) = y'(0) = y''(0) = y'''(0) = 0$.

③ Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ by transform method,

where $y(0) = 0,\ y'(0) = 1$.

④ Solve $\frac{d^2y}{dt^2} + y = 6\cos 2t$, where $y'(0) = 1,\ y(0) = 3$

⑤ Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t,\ y = \frac{dy}{dt} = 0$ when $t = 0$

⑥ Solve $t\frac{d^2y}{dt^2} + (1-2t)\frac{dy}{dt} - 2y = 0,\ y(0) = 1,\ y'(0) = 2$

⑦ Solve $t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \sin t$, when $y(0) = 1$

⑧ Solve → $t\frac{d^2y}{dt^2} + (t-1)\frac{dy}{dt} - y = 0,\ y(0) = 5,\ y(\infty) = 0$

⑨ Solve the following simultaneous equation

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$$

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$$

when $x(0) = 3,\ y(0) = 0$

10 Solve $\frac{d^4y}{dt^4} - k^4y = 0$ by Laplace transform Method

when $y(0) = 1,\ y'(0) = y''(0) = y'''(0) = 0$

Chapter-7 Fourier Transforms

① If $F(s)$ is the fourier transform of $f(x)$, then show that $e^{ias}F(s)$ is the fourier transform of $f(x-a)$

② If $F(s)$ is the infinite fourier transformation of $f(x)$, then Fourier transform of $f(x)\cos ax$ is given by $\frac{1}{2}F(s-a)+\frac{1}{2}F(s+a)$

③ Find the fourier transform of the function

$$f(x)=\begin{cases} k, & \text{if } 0<x<a \\ 0, & \text{otherwise} \end{cases}$$

④ Find the fourier transform of the function

$$f(x)=\begin{cases} -1, & -a<x<0 \\ 1, & 0<x<a \\ 0, & \text{otherwise} \end{cases}$$

⑤ Find the fourier transform of the function

$$f(x)=\begin{cases} xe^{-x}, & x>0 \\ 0, & x<0 \end{cases}$$

⑥ Find the fourier transform $f(x)$, if

$$f(x)=\begin{cases} \frac{1}{2\varepsilon}, & |x|\le\varepsilon \\ 0, & |x|>\varepsilon \end{cases}$$

⑦ Find the complex fourier transform of $e^{-|x|}$

(8) Find the fourier sine transform of $\frac{e^{-ax}}{x}$

(9) Find the fourier cosine transform of e^{-x^2}

(10) Find the fourier cosine transform of $\frac{1}{1+x^2}$ and deduce the sine transform of $\frac{x}{1+x^2}$

(11) Find the sine transform of the function

$$f(x) = \begin{cases} \sin x , & 0 < x < a \\ 0 , & x > a \end{cases}$$

(12) Find the fourier sine and cosine transform of the function x^{m-1}

(13) Find the fourier sine and cosine transform of e^{-ax}

(14) Find $f(x)$ if its fourier sine transform is $\frac{s}{1+s^2}$

(15) Find the fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$

(16) Find the fourier transform of

$$f(x) = \begin{cases} 1 - x^2 , & |x| < 1 \\ 0 , & |x| > 1 \end{cases}$$

and hence evaluate $\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3} \right) \cos \frac{x}{2} dx$

(17) Find the fourier transform of $f(x)$ defined by $f(x) = \begin{cases} 1 , & |x| < a \\ 0 , & |x| > a \end{cases}$ and hence evaluate

$$\int \frac{\sin sa \cdot \cos sx}{s} dx$$

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(16) Find $f(x)$ if its cosine transform is $\frac{1}{1+s^2}$

(17) Solve the integral equation $\int_0^\infty f(x) \cos sx \, dx = e^{-s}$

(18) State & prove convolution theorem for fourier transform

(21) Define relation between fourier and Laplace transform

(20) Define Parseval's identities for fourier sine and cosine transform

(22) find the fourier sine and cosine transform of the function e^{-mx}, $m>0$ by using its second derivati

(24) Using Parseval's identity, prove that

$$\int_0^\infty \frac{x^2 \, dx}{(x^2+1)^2} = \frac{\pi}{4}$$

(25) Using Parseval's identity, prove that

$$\int_0^\infty \frac{\sin ax}{x(a^2+x^2)} \, dx = \frac{\pi}{2} \left(\frac{1-e^{-a^2}}{a^2} \right)$$

(26) Using Parseval's identity show that

$$\int_0^\infty \frac{dx}{(x^2+25)(x^2+81)} = \frac{\pi}{1260}$$

(27) Find the finite fourier sine and cosine transform of $f(x) = 1$

(28) Find the finite sine transform of $f(x) = 2x$, where $0<x<4$

29) Find the finite fourier sine transform of $\cos ax$ where $0 < x < \pi$

30) Find the finite cosine transform of $f(x)$ if

$$f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ -1, & \frac{\pi}{2} < x < \pi \end{cases}$$

31) Show that the finite sine transform of $\frac{x}{\pi}$ is $(-1)^{s+1}\frac{1}{s}$.

32) Find the finite cosine transform of $f(x) = \sin ax$

33) Find the finite cosine transform of $f(x)$ if

$$f(x) = -\frac{\cos k(\pi - x)}{k \sin k\pi}$$

34) Find the fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \left[\frac{x\cos x - \sin x}{x^3}\right] \cos\frac{x}{2}\, dx = -\frac{3\pi}{16}$

35) Let $f(t) = \begin{cases} 1 - \frac{t}{a} & \text{for } 0 < t < a \\ 1 + \frac{t}{a} & \text{for } -a < t < 0 \\ 0 & \text{otherwise} \end{cases}$, find $F\{f(t)\}$

36) Find the fourier sine and cosine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$$

37) ~~Find~~ If the fourier transform of $f(x)$ is $\bar{f}(s)$, then show that fourier transform of $f(ax)$ is $\frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$

38. Find the finite cosine transform of f

$$f(x) = -\frac{\cos k(\pi - x)}{k \sin k\pi}$$

Chapter-10 (Solution of Differential Equation by Fourier Transforms)

① The temperature u in a semi-infinite rod is determined by $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 \leq x < \infty$

with the conditions →

(i) $u = 0$ when $t = 0$, $x > 0$

(ii) $\frac{\partial u}{\partial x} = -\mu$ when $x = 0$

(iii) $\frac{\partial u}{\partial x} \to 0$ as $x \to \infty$

Determine the temperature formula.

② Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, given that

(i) $u(0,t) = 0$, (ii) $u(\pi,t)=0$, (iii) $u(x,0) = 2x$, when $0 < x < \pi$, $t > 0$

③ Using the fourier sine transform, solve the partial differential equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ with the boundary condition

(i) $u = u_0$ when $x = 0$, $t > 0$ and the initial condition

(ii) $u = 0$ when $t = 0$, $x > 0$

④ The initial temperature of an infinite bar is given by

$$\theta(x) = \begin{cases} \theta_0 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

Determine the temperature at any point x and at any instant t.

⑤ Solve $\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$, if (i) $u(0,t) = 0$

(ii) $u(x,0) = e^{-x}$, $x > 0$ (iii) $u(x,t)$ is bounded , when $x > 0, t > 0$

⑥ Using suitable transform, solve the differential equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial t} , \quad 0 \leq x \leq \pi \text{ and } t > 0$$

where $V(0,t) = 0 = V(\pi,t)$ and $V(x,0) = V_0$ Constant

Chapter - 8

(10) The co-ordinate (x, y) of a particle moving along in plane curve at any time t is given by

$$\frac{dy}{dt} + 2x = \sin 2t$$

$$\frac{dx}{dt} - 2y = \cos 2t$$

If at $t=0$, $x=1$ and $y=0$, show by using transforms that particle moves along the curve $4x^2+4xy+5y^2=4$

(11) Solve $\rightarrow$ $\frac{dx}{dt} = 5x+y$, $\frac{dy}{dt} = x+5y$ when $x(0)=-3, y(0)=7$

(12) ~~Solve~~ $\cancel{\frac{d^2y}{dt^2}}$

12. Find the following using Parseval's identity!

$$\int_0^\infty \frac{x^2}{(x^2+1)^2}\,dx .$$