B.Sc/BA (I)

Ordinary Differential Equations

UNIT-I

1) To find the necessary and sufficient conditions that the equation $Mdx + Ndy = 0$ may be exact.

2) Solve the differential equation
$$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$

3) Solve
$$(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$$

4) Solve
$$(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$$

5) Solve
$$(x^2 + y^2 + 2x)dx + 2ydy = 0$$

6) Solve
$$(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$$

7) Solve
$$x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + 2y^2 - x^2 = 0$$

8) Solve
$$p^3 - p(x^2 + xy + y^2) + xy(x+y) = 0$$

9) Solve $y = -px + x^4 p^2$

10) Solve $y = 2p + \sqrt{1+p^2}$

11) Solve $y = 2px + y^2 p^3$

12) Solve $p^3 - 4xyp + 8y^2 = 0$

13) Solve $p = \log(px - y)$

14) Solve $\sin px \cos y = \cos px \sin y + p$

15) Solve and find complete primitive and singular solution of the equation

(i) $3y = 2px - \frac{2p^2}{x}$

(ii) $x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} + 4x = 0$

16) Define Langrange's and Clairaut's equation.

$\boxed{\text{UNIT} - \text{II}}$

Q-1 Define Orthogonal trajectory.

Q-2 Find the orthogonal trajectories of

(i) $x^2 + y^2 + 2gx + c = 0$ where g is parameter & c is constant

(ii) $r^n \sin n\theta = a^n$

(iii) ~~$x^{\frac{2}{3}}$~~ $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

Q-3 Show that system of confocal

conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self-orthogonal

Q-4 Solve the differential equations

(i) $(D^4 + 5D^2 + 6)y = 0$

(ii) $\frac{d^4y}{dx^4} + a^4y = 0$

(iii) $\frac{d^4y}{dx^2} + a^2y = \sec ax$

(iv) $\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}$

(v) $\frac{d^3y}{dx^3} - 5\frac{dy^2}{dx^2} + 7\frac{dy}{dx} - 3y = e^{2x}\cosh x$

(vi) $\frac{d^2y}{dx^2} + y = \sin x \sin 2x$

(vii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$

~~(viii)~~ $\cancel{\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = x^2e^x}$

(viii) $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x.\cos 2x$

(ix) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x}\sin 2x$

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Q-5 Solve the Differential equation

(i) $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$

(ii) $(x^2D^2 - 3xD + 5)y = \sin(\log x)$

(iii) $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

$\boxed{\text{UNIT-3}}$

Q-1 Solve the differential equation

(i) $x^2\frac{d^2y}{dx^2} - (x^2+2x)\frac{dy}{dx} + (x+2)y = x^3e^x$

(ii) $(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$

Q-2 Solve $\sin^2x\,\frac{d^2y}{dx^2} = 2y$, given that $y = \cot x$ is a solution.

Q-3 Solve by removing first derivative (i) $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = 0$

(ii) $\frac{d}{dx}\left(\cos^2x\,\frac{dy}{dx}\right) + y\cos^2x = 0$

Q-4 Solve by changing independent variable

(i) $\cos x \dfrac{d^2y}{dx^2} + \sin x \dfrac{dy}{dx} - 2y \cos^3 x = 2\cos^5 x$

(ii) $\dfrac{d^2y}{dx^2} + \dfrac{2}{x}\dfrac{dy}{dx} + \dfrac{a^2}{x^4} y = 0$

Q.5 Solve by method of variation of parameter.

(i) $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$

(ii) $x^2 \dfrac{d^2y}{dx^2} - 2x(1+x)\dfrac{dy}{dx} + 2(1+x)y = x^3$

Q6 Solve by method of undetermined coefficients.

(i) $(D^2 - 2D + 3)\, y = x^3 + \sin x$

(ii) $\dfrac{d^2y}{dx^2} - \dfrac{2dy}{dx} = e^x \sin x$

(iii) $(D^2 + 9)\, y = x^2 \cos 3x$

UNIT-4

Q-1 Solve the simultaneous equations

(i) $\dfrac{dx}{dt} + 5x + y = e^t$ & $\dfrac{dy}{dt} - x + 3y = e^{2t}$

(ii) $\frac{dx}{dt} + 2y\frac{dy}{dt} - 2x + 2y = 3e^{t}$

$3\frac{dx}{dt} + \frac{dy}{dt} + 2x + y = 4e^{2t}$

Q-2 Solve the simultaneous equations

(i) $\frac{a\,dx}{(b-c)} = \frac{b\,dy}{(c-a)zx} = \frac{c\,dz}{(a-b)xy}$

(ii) $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z+\frac{1}{z}}$

(iii) $\frac{dz}{z(x+y)} = \frac{dy}{z(x-y)} = \frac{dz}{x^2+y^2}$

(iv) $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dz}{z(xy-2x^2)}$

(v) $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{2x-3y}$

Q-3 Solve the differential equations

(i) $yz\log z\,dx - zx\log z\,dy + xy\,dz = 0$

(ii) $2yz\,dx + zx\,dy - xy(1+z)\,dz = 0$

(iii) $(yz+z^2)\,dx - xz\,dy + xy\,dz = 0$

(iv) $(x-3y-z)\,dx + (2y-3x)\,dy + (z-x)\,dz = 0$

(v) $z(z-y)\,dx + (z+x)z\,dy + x(x+y)\,dz = 0$