Some Important questions of Linear Algebra

CH-1

Q1) Show that the set $Q(\sqrt{2}) = \{a+b\sqrt{2} \mid a,b \in Q\}$ is Vector Space over Q w.r.t. composition of addition and Scaler mult

Q2). Show that any Plane passing through the origin is a subspace of R^3.

Q3) Let V be the vector space of all square matrices over R. Determine which of the following are subspaces of V.

(i) $W = \{A : A \in V$ and A is Singular$\}$ (ii) $W = \{A \mid A \in V, A^2 = A\}$

Q4) Let V be the space of all functions from R to R and W_1 and W_2 be the subspaces of V. Define by

$$W_1 = \{f : f(x) = f(-x) \text{ for all } x \in R\}$$
$$W_2 = \{f : f(-x) = -f(x) \text{ for all } x \in R\}$$

Show that $V = W_1 \oplus W_2$

CH-2

Q5) Show that if two vectors of a vector space $V(F)$ are L.D., then one of them is a scaler multiple of the other.

Q6. In the vector space $R^3(R)$, let $u = (1,2,1)$, $v = (3,1,5)$ and $w = (3,-4,7)$. Prove that the subspace spanned by $S = \{u, v\}$ and $T = \{u, v, w\}$ are the same

Q7) Show that the set $\{(2,1,4), (1,-1,2), (3,1,-2)\}$ forms a basis of R^3.

Q8) Extend the set of vectors $\{(0,1,2), (3,-1,4)\}$ to form a basis of R^3.

Q9). If W_1 and W_2 are subspaces of V, $\dim W_1 = 4$, $\dim W_2 = 5$ and $\dim V = 7$, then find the Possible value of $\dim(W_1 \cap W_2)$.

CH-3

Q10) If W is a subspace of $V = V_2(R)$ generated by $(1,2)$, find V/W and its basis.

Q11) If W is a subspace of $V_3(R)$ generated by $\{(1,0,0), (1,1,0)\}$, find V/W and its basis.

Q12) Let V be the vector space of all 2×2 matrices over C and W_1, W_2 be its two subspaces given by
$W_1 = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} ; a,b,c \in C \right\}$ and $W_2 = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} ; a,b \in C \right\}$

Prove that $\dim\left(\frac{W_1+W_2}{W_2}\right) = \dim\left(\frac{W_1}{W_1 \cap W_2}\right)$

Q14)

Q13) Show that the function $T: R^2 \to R^3$ given by $T(x,y) = (x+y, x-y, y)$ is a linear transformation.

Q14) Show that the transformation $T: R^3 \to R^2$ defined by $T(x_1, x_2, x_3) = (x_1, x_2)$ is a L.T. and is onto but not one-one.

Q15) Show that L.T. $T: R^2 \to R^2$ defined by $T(x_1, x_2) = (x_1\cos\theta + x_2\sin\theta, -x_1\sin\theta + x_2\cos\theta)$ is a V.S. isomorphism.

Q16) Let $u_1 = (1,1)$, $u_2 = (0,1)$ be a basis of R^2. Let $T: R^2 \to R$ be the L.T. for which $T(u_1) = 3$ and $T(u_2) = -2$.
Find the L.T. 'T'.

Q17) Find a L.T. $T: R^3 \to R^2$ such that $T(1,1,1) = (1,0)$ and $T(1,1,2) = (1,-1)$. Also, verify your answer.

Q18) Find the L.T. $T: R^3 \to R^3$ such that $T(X) = AX$,
where $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

Q19) Find the L.T. $T: P_3(x) \to P_3(x)$ such that $T(1+x) = 1+x$,
(H5) $T(2+x) = x+3x^2$ and $T(x^2) = 0$.

Q20) For the L.T. $T: R^2 \to R^3$ such that $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$
Find the basis and dim. of its range space and its Null space. Also verify Rank-Nullity Thm.

Q21). If $T: R^4 \to R^3$ is a L.T. defined by $T(e_1) = (1,1,1)$
$T(e_2) = (1,-1,1)$, $T(e_3) = (1,0,0)$, $T(e_4) = (1,0,1)$.
then verify that $\rho(T) + \nu(T) = \dim R^4 = 4$.

Q22) Find a L.T. $T: R^3 \to R^4$ whose null space is generated by $(0,1,-3)$, $(0,-3,4)$

Q23) Find the L.T. $T: R^3 \to R^4$ whose range is generated by $(1,2,0,-4)$ and $(2,0,-1,-3)$.

(H.6)

Q24) Given the L.T. $Y = AX$ where $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ -2 & 3 & 5 \end{bmatrix}$

Show that

(i) it is singular

(ii) the images of L.I. vectors $X_1 = (1,1,1)$, $X_2 = (2,1,2)$, $X_3 = (1,2,3)$ are L.D.

Q25) Prove that the L.T. defined by $T(x,y) = (ax+by, cx+dy)$ where $a,b,c,d \in R$, is Non-singular transformation iff $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

Q26) If $T: R^3 \to R^3$ be a linear operator defined by $T(x,y,z) = (x+z, x-z, y)$. Show that T is a invertebee and find T^{-1}.

(H.7)

Q27) Find the matrix representing the transformation $T: R^3 \to R^4$ defined by $T(x,y,z) = (x+y+z, 2x+z, 2y-z, 6y)$ relative to the standard basis of R^3 and R^4.

Q28) If the matrix of a L.T. $T: R^3 \to R^3$ relative to ordered basis $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ is

$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$: Find the matrix of T relative to the basis $B' = \{(0,1,-1), (-1,1,0), (1,-1,1)\}$.

Q29) Let $P_n(t)$ be the V.S. of all Polynomials of deg n. Let $D: P_3(t) \to P_2(t)$ be the L.T. defined by $D(P(t)) = P'(t)$. Find the Mtx. of D w.r.t. Standard basis $\{1, t, t^2, t^3\}$ and $\{1, t, t^2\}$.

Q30) Let $T: R^3 \to R^2$ be a L.T. $T(x,y,z) = (2x+y-z, 3x-2y+4z)$

Find Mtx. of T w.r.t. ordered basis $B_1 = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $B_2 = \{(1,3), (1,4)\}$ of R^3 & R^2 resp.

Also verify $[T : B_1, B_2][e, B_1] = [T(4), B_2]$

(Ch-9)

Q31) Find eigen values and eigen vectors for the Mtx

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q32) Let V be the space of all real valued continuous functions. Let T be a linear operator i.e $T: V \to V$ s.t. $(Tf)(x) = \int_0^x f(t)\,dt$. Prove that T has No eigen values.

Q33) For the L.O. $T: R^3 \to R^3$, find the eigen values and the basis for eigen space, when $T(x,y,z) = (x+y+z, 2y+z, 2y+3z)$

Q34) Show that mtx. $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is Not diagonalizable over the field C.

Q35) Let $T: R^3(R) \to R^3(R)$ be a L.T. defined by $T(x,y,z) = (2x-y, x+y+z, 2z)$. Find chart

(Ch-10) and minimal Polynomial of T.

Q36) Show that $\langle u, v \rangle = 2x_1\bar{y}_1 + x_1\bar{y}_2 + x_2\bar{y}_1 + x_2\bar{y}_2$, defines an Inner Product on $V_2(C)$ where $u = (x_1, x_2)$, $v = (y_1, y_2) \in V_2(C)$.

Q37) Prove that every Inner Product space is a normed linear space but converse is NOT true.

Q38) Obtain an orthonormal basis with respect to standard inner Product for the subspace of R^3 generated by $\{(1,0,1), (1,0,-1), (0,3,4)\}$

Q39) Let W be a subspace of $R^4(R)$ generated by the vectors $u_1 = (1,2,3,-2)$, $u_2 = (2,4,5,-1)$ obtain a basis for $W^\perp$.

Q40). Using Gram-Schmidt Process, find an orthonormal basis of the subspace W of $V_3(C)$ Spanned by $u_1 = (1,0,i)$, $u_2 = (2,1,1+i)$