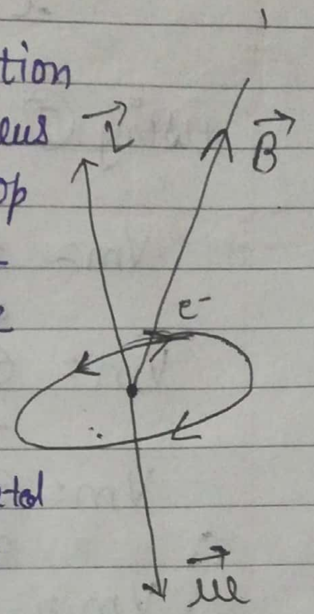
Atom in an field

Quantum Theory of Normal Zeeman Effect :→



In an atom, the orbital motion of electron around the nucleus is equivalent to a current loop and the (magnetic moment of current loop is given by) behave like a magnetic dipole.

We know that the ratio of magnetic moment ($\vec{\mu_L}$) associated with the orbital motion of electron to its angular momentum ($\vec{L}$) is given by

$$\frac{\vec{\mu_L}}{\vec{L}} = \frac{-e}{2m} \longrightarrow (1)$$

$$\vec{\mu_L} = \frac{-e}{2m}\vec{L} \longrightarrow (2)$$

-ve sign indicate that direction of μ_L is opposite to $\vec{L}$.

when magnetic field $\vec{B}$ is applied to this dipole. in that case torque is acting on it. We know the Potential energy of this dipole

$$V_m = -\mu_L \vec{B} \cos\theta \longrightarrow (3)$$

using eqn (2) and (3)

$$V_m = -\left(\frac{-e}{2m}\right) L \vec{B} \cos\theta$$

$$\boxed{V_m = \frac{e}{2m} L \vec{B} \cos\theta} \longrightarrow (4)$$

Now $L = l^* \hbar$ → (5)

where $l^* = \sqrt{l(l+1)}$

where l = Orbital quantum no.

$l = 0, 1, 2 \ldots \ldots\ldots$

using (5) in (4)

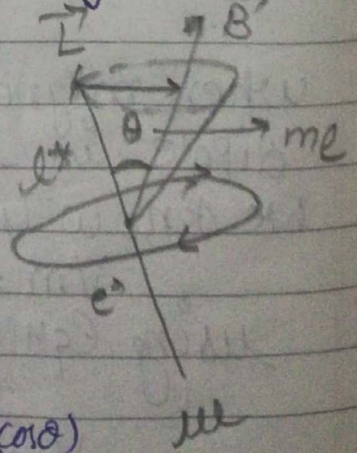
$$V_m = \frac{e}{2m} \vec{B} (l^* \hbar) \cos\theta$$

$$V_m = \frac{e}{2m} \vec{B} \, l^* \frac{h}{2\pi} \cos\theta \quad \left[\hbar = \frac{h}{2\pi}\right]$$

$$V_m = \frac{e}{2m} B \frac{h}{2\pi} (l^* \cos\theta)$$

$$V_m = \frac{eh}{4\pi m} B (l^* \cos\theta) \rightarrow (6)$$

Now we applied a magnetic field on a magnetic dipole acc to larmor th^m L vector is precesses about the field (Similar to the precession of mechnical top in gravitational field). Now we take a Projection of $\vec{L}$ vector in dir^n of field $\vec{B}$ it denoted by m_l. Now angle in b/w $\vec{L}$ and $\vec{B}$ is θ.



$$\cos\theta = \frac{m_l}{l^*}$$

$$m_l = l^* \cos\theta$$

Now put in eq^n (6) value $(l^* \cos\theta)$

$$V_m = \frac{eh}{4\pi m} \vec{B} m_l$$

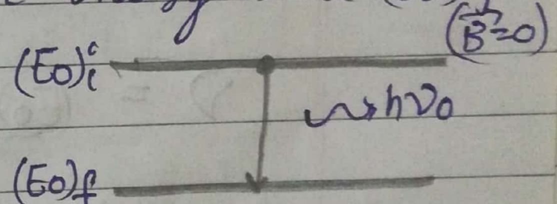
$$\boxed{V_m = \mu_B \vec{B} m_l} \longrightarrow (7)$$

where $\boxed{\mu_B = \frac{eh}{4\pi m}}$ Bohr magneton (*)

If we placed a magnetic dipole (electron orbit) in a magnetic field Now in that case it stored P.E is $(V_m = \mu_B B m_l)$.

If no magnetic field is applied $(\vec{B}=0)$ we have Two energy level higher energy level value $(E_0)_i$ & lower energy level $(E_0)_f$

The frequency emitted by Spectral line



$$h\nu_0 = (E_0)_i - (E_0)_f$$

$$\boxed{\nu_0 = \frac{(E_0)_i - (E_0)_f}{h}} \longrightarrow (8)$$

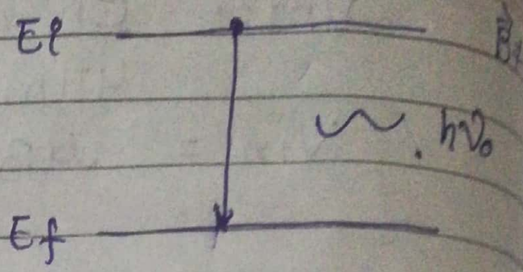
$(\vec{B}=0)$ Now in Case of Zn singlet or Zn emission spectrum is single having their frequency is $\nu_0 = \frac{(E_0)_i - (E_0)_f}{h}$

Now, ∵ magnetic field $\vec{B}$ is applied, Total energy of atom is given by

$[\because V_m = \mu_B \vec{B} m_l]$

$$E = E_0 + V_m$$
$$E = E_0 + \mu_B B m_l$$

$$\left.\begin{aligned} E_i &= (E_0)_i + \mu_B B (m_e)_i \\ E_f &= (E_0)_f + \mu_B \vec{B} (m_e)_f \end{aligned}\right] \quad (9)$$



Now, if electron jump from higher energy level E_i to lower energy level E_f, the frequency of emitted spectral line

$$\nu = \frac{E_i - E_f}{h} = \frac{(E_0)_i + \mu_B B (m_e)_i - (E_0)_f - \mu_B B (m_e)_f}{h}$$

$$\nu = \frac{(E_0)_i - (E_0)_f + \mu_B B[(m_e)_i - (m_e)_f]}{h}$$

$$\nu = \frac{(E_0)_i - (E_0)_f}{h} + \frac{\mu_B B[(m_e)_i - (m_e)_f]}{h} \quad (10)$$

put value (8) & (7) in eq^n (10)

$$\nu = \nu_0 + \frac{eh}{4\pi m}\frac{B}{h}[(m_e)_i - (m_e)_f]$$

$$\nu = \nu_0 + \Delta m_e \left(\frac{eB}{4\pi m}\right)$$

$$\boxed{\nu = \nu_0 + (\Delta m_e)\frac{eB}{4\pi m}} \longrightarrow (11)$$

from slection rules;
Only those transitions are allowed
$\Delta m_l = 1, 0, -1$

Three transition allowed
Now put in eq^n (ii) Δm_l value

$\nu_1 = \nu_0 + \frac{eB}{4\pi m}$ $(\Delta m_l = 1)$

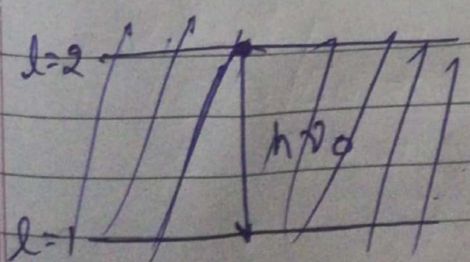
$\nu_2 = \nu_0$ $(\Delta m_l = 0)$

$\nu_3 = \nu_0 - \frac{eB}{4\pi m}$ $(\Delta m_l = -1)$

Now, we can write as $\boxed{\frac{eB}{4\pi m} = \Delta\nu}$ in above eq^n

$\nu_1 = \nu_0 + \Delta\nu$
$\nu_2 = \nu_0$
$\nu_3 = \nu_0 - \Delta\nu$

Hence in case is Zinc single (Zn) a single spectral line is split into three component lines having frequency is ν_1, ν_2 & ν_3 in presence of magnetic field.



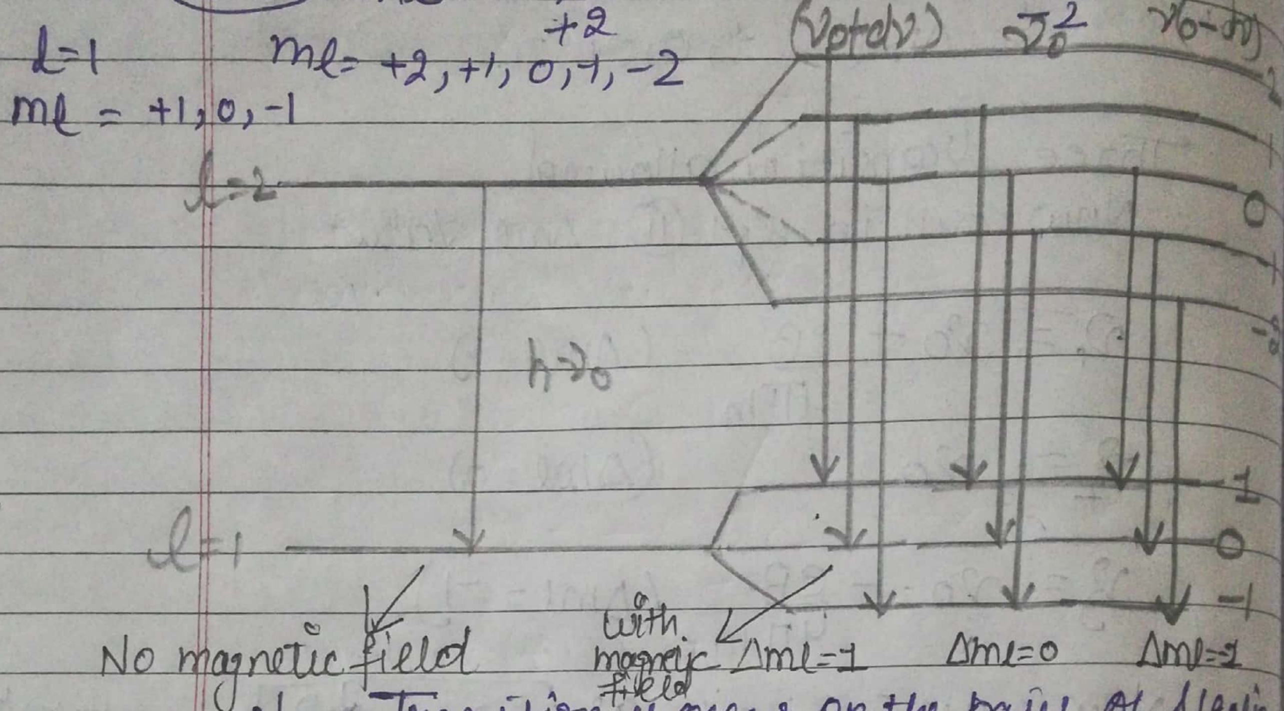
Since m_l can have $(2l+1)$ values i.e a given state can split upto $(2l+1)$ values.

$l=2$ $m_l = +l \; \dots \dots \; -l$

$m_l = +2, +1, 0, -1, -2$

$l=1$

$m_l = +1, 0, -1$



Now Transitions occur on the basis of selection rule $\Delta m_l = 0, \pm 1$

Single spectral line splitted into 3 lines in presence of external magnetic field

$\Delta m_l = \pm 1$ σ lines

$\Delta m_l = 0$ π lines

2σ lines and 1π lines

Zeeman Shift

for normal zeeman shift, the spin angular momentum, $\vec{S} = 0$, so $s = 0$ Singlet State

$$2S+1 = 1$$
$$2S = 0$$
$$S = 0, s = 0 \quad \text{Net spin} = 0$$

In this case $j = l + s$

$$j = l + 0 = l \qquad j = l$$

$$m_j = m_l + m_s$$
$$\boxed{m_j = m_l}$$

Lande g factor

$$g = 1 + \frac{j(j+1) - l(l+1) + s(s+1)}{2j(j+1)}$$

Now $g = 1 + \dfrac{l(l+1) - l(l+1) + 0(0+1)}{2l(l+1)}$

$$g = \frac{2l(l+1) + l(l+1)}{2l(l+1)} = \frac{2l(l+1)}{2l(l+1)}$$

$$g = 1$$

Also, during Normal zeeman effect

$$\nu_1 = \nu_0 + \frac{eh}{4\pi m} \quad \text{for } (\Delta m_l = 1)$$

$$\nu_2 = \nu_0 \quad \text{for } (\Delta m_l = 0)$$

$$\nu_3 = \nu_0 - \frac{eh}{4\pi m} \quad \text{for } (\Delta m_l = -1)$$

change in frequency

$$\Delta\nu = \pm \frac{eB}{4\pi m} \longrightarrow *$$

The frequency of spectral line having wavelength λ is given by

$$\nu = \frac{c}{\lambda}$$

$$d\nu = -\frac{c}{\lambda^2} d\lambda \longrightarrow **$$

Compare * & **

$$\pm \frac{eB}{4\pi m} = -\frac{c}{\lambda^2} d\lambda$$

$$\boxed{d\lambda = \pm \frac{\lambda^2}{c} \frac{eB}{4\pi m}}$$

Where $d\lambda \pm \frac{\lambda^2}{c}$ is a Zeeman shift