Ch-1 (Topology of Real Numbers)

Ques1 Every non empty subset of real numbers which is bounded below has a real number as its infimum

Ques2 Between two distinct real numbers, there are infinitely many rational numbers

Ques3 if S and T are non empty bounded subsets of R, then prove that $S \cup T$ is also bounded and $\sup(S \cup T) = \max\{\sup S, \sup T\}$

Ques4 Prove that set of rationals is not order complete.

Ques5 Let $A = \{x : x \in Q^+ \text{ and } x^2 > 2\}$. prove that set A has no glb in Q.

Ques6 prove that the union of an arbitrary family of open sets is an open set.

Ques7 prove that the intersection of a finite number of open sets is an open set. ~~and~~

Ques8 The intersection of an arbitrary family of open set may not be an open set. ~~and~~ give an example of it.

Ques9 For any set A, A° is open

Ques10 The interior of set A is largest open subset of A

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Ques 11 The union of finite number of closed set is a closed set

Ques 12 Give an example that the arbitrary union of closed sets may not be closed set

Ques 13 Is every infinite set open? Justify your answer

Ques 14 What is limit point and if A and B of R then
i) $A \subset B \Rightarrow A' \subseteq B'$
(ii) $(A \cup B)' = A' \cup B'$
and show that by an example that $(A \cap B)' \neq A' \cap B'$

Ques 15 State and prove Bolzano - Weierstrass Theorem

Ques 16 The derived set of any set is a closed set

Ques 17 Prove that the supremum of a non empty bounded set is either the greatest member of the set or is a limit point of the set.

Ques 18 Closure of a set is a closed set

Ques 19 The closure of a set $A \subseteq R$ is the smallest closed superset of A.

Ques 20 if A and B are arbitrary subsets of R then
(i) $A \subset B \Rightarrow \overline{A} \subseteq \overline{B}$
(ii) $\overline{(\overline{A})} = \overline{A}$

Ques 21 Every open cover of a compact set has a finite subcover.

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Ques 22 Every set satisfying the Heine Borel property is a compact set

Chapter - 2 (Sequences)

Ques 1 prove that every convergent sequence is bounded but not conversely.

Ques 2 Define convergent, Divergent and Oscillating sequence and give an example of each.

Ques 3 Discuss the boundness of sequences $\langle a_n \rangle$
(i) $a_n = \frac{2n+3}{3n+4}$ (ii) $a_n = \left(1+\frac{1}{n}\right)^n$

Ques 4 Show that the sequence $\langle a_n \rangle$ where $a_n = x^n$ converges to 0 if $|x| < 1$

Ques 5 Let $\langle a_n \rangle$ be a sequence such that $a_n \neq 0$ for all n and $\frac{a_{n+1}}{a_n} \to l$ if $|l| < 1$ then $\lim_{n\to\infty} a_n = 0$

Ques 6 State and prove squeeze Principle
Ques 7 State and prove Cauchy's First theoram on limit
Ques 8 if $\langle a_n \rangle$ is a sequence of +ve terms converging to $a > 0$ and $b_n = (a_1 \cdot a_2 \cdots a_n)^{1/n}$, then $\langle b_n \rangle$ converges to a

Ques 9 State and Prove Cauchy's second theoram on limits
Ques 10 Show that $\lim_{n\to\infty} \left[\frac{1}{n^2} + \frac{1}{(n+1)^2} + \cdots + \frac{1}{(2n)^2}\right] = 0$

Ques 11 Show that the sequence $\langle n^{1/n} \rangle$ converges to limit
Ques 12 Show that $\lim_{n\to\infty} \left(\frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdots \frac{n}{n-1}\right)^{\frac{1}{n}} = 1$

Ques 13 if $\langle a_n \rangle$ converges to l_1 and $\langle b_n \rangle$ converges to l_2, then

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sequence $\langle \frac{c_n}{n} \rangle$ converges to $l_1 l_2$ where
$c_n = a_1 b_n + a_2 b_{n-1} + \cdots + a_n b_1$

Ques 14 Using Cauchy second theoram on limits
$$\lim_{n \to \infty} \left[\left(\frac{2}{1}\right) \left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^3 \cdots \left(\frac{n+1}{n}\right)^n \right]^{\frac{1}{n}} = e \text{ and}$$

hence $\lim_{n \to \infty} \left(\frac{n^n}{n!}\right)^{1/n} = e$

Ques 15 prove that a monotonically increasing sequence $\langle a_n \rangle$ which is bounded above converges to its least upper bound

Ques 16 prove that a monotonically decreasing sequence $\langle a_n \rangle$ which is bounded below converges to its greatest lower bound

Ques 17 State and prove Nested Interval property (or Cantor Intersection theoram)

18 Discuss the convergence of the sequence $\langle a_n \rangle$ where
$$a_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}$$

19 prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ exists and lie between 2 & 3

20 prove that the sequence $\langle a_n \rangle$ defined by $a_1 = 1$ and $a_n = \sqrt{2 + a_{n-1}}$ converges to the positive root of the equation $x^2 - x - 2 = 0$

21 prove that the sequence $\langle a_n \rangle$ defined by $a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}$ is convergent and
$$2 \le \lim_{n \to \infty} a_n \le 3$$

22 Every bounded sequence has a cluster point

23 Define Cauchy's sequence. Prove that every Cauchy's sequence is bounded. Is the converse true? if not show by an example.

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24 A sequence converges if and only if it is a Cauchy's sequence.

25 Show that the sequence $\langle a_n \rangle$ defined by

$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$ does not converge

26 Define subsequence. A real number l is a limit point of a sequence $\langle a_n \rangle$ iff there exist a subsequence of $\langle a_n \rangle$ converging to l.

Ch-3 (Infinite series)

Ques 1 Show that the series $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots$ converges

Ques 2 Show that the series $1^2 + 2^2 + 3^2 + \cdots + n^2 + \cdots$ diverges to ∞

Ques 3 State and prove that Cauchy's General Principle of Convergence

Ques 4 If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. Is converse true? If not show by an example

Ques 5 State and prove General test for the convergence of positive term series.

Ques 6 If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two series of positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = l$ (finite and non zero) then both the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or diverge together.

Ques 7 State and prove Hyper Harmonic series

Ques 8 Show that the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ is dgt.

Ques 9 Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{n^2}\right)^{\frac{1}{n}}$ is divergent

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Ques 10 Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

Ques 11 Test the convergence of series $\sum_{n=1}^{\infty} \sqrt{n^4+1} - \sqrt{n^4-1}$

Ques 12 Test the Convergence of series $\frac{1}{1+x} + \frac{x}{1+x^2} + \frac{x^2}{1+x^3} + \dots$

where $x>0$

Ques 13 Examine the convergence of series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$

Ch-4 (Infinite series)

Ques 1 State and prove D'ALEMBERT'S RATIO TEST

Ques 2 Test for convergence of the series

$$\frac{1}{1.2.3} + \frac{x}{4.5.6} + \frac{x^2}{7.8.9} + \dots \quad (x>0)$$

Ques 3 Test the following series for the convergence

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \dots \quad x>0$$

Ques 4 State and prove Cauchy's Root Test

Ques 5 Discuss the convergence of the series

$$\frac{1}{2} + \left(\frac{2}{3}\right)x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \dots$$

Ques 6 State and prove Raabe's Test

Ques 7 State and prove Logarithmic Test

Ques 8 Test the convergence of the series

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \dots$$