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DEGENERACY AND
BOSE EINSTEIN CONDENSATION

DEGENERACY:

Deviation in the Behaviour of the B.E gas from that of perfect gas is called degeneracy.

As we know;

$$A = e^{-\alpha} = \frac{n}{V}\left(\frac{h^2}{2\pi mkT}\right)^{3/2}$$

so As the temp decreases or the particle density increases value of A increases showing the behaviour of a perfect gas depart from that of a classical perfect gas as we derived it assuming it was a perfect classical gas.

$\therefore$ **Degree of degeneracy → large**

When:
- → temp is LOW
- → particle density is HIGH

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So we know, total no. of particles in a B.E gas is

$$n = \frac{V}{h^3}(2\pi m KT)^{3/2}\left[A + \frac{A^2}{2^{3/2}} + \frac{A^3}{3^{3/2}} + \cdots\right]$$

(derived in last class)

$$\frac{n}{V} = \left(\frac{2\pi m KT}{h^2}\right)^{3/2}\left[A + \frac{A^2}{2^{3/2}} + \frac{A^3}{3^{3/2}} + \cdots\right]$$

POINT TO NOTE:

As α can never be negative

$$\boxed{\text{so } e^{-\alpha}\underset{\text{energy state}}{(\text{lowest})} = A = 1}$$

$\therefore$ maximum particle density $\left(\frac{n}{V}\right)$ will be obtained for $A=1$. **(Max. value of A). (at lowest α)**

$\Rightarrow$

$$\left(\frac{n}{V}\right)_{max} = \left(\frac{2\pi m KT}{h^2}\right)^{3/2}\underbrace{\left[1 + \frac{1}{2^{3/2}} + \frac{1}{3^{3/2}} + \cdots\right]}_{\zeta\left(\frac{3}{2}\right)}$$

$$\left(\frac{n}{V}\right)_{max} = \left(\frac{2\pi m KT}{h^2}\right)^{3/2} \times 2.612 \quad \star \text{ — } (1)$$

★ In Mathematics $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} = \zeta(3/2)$

$= 2.612$

Source: (RIEMANN ZETA FUNCTION)

This case ($A=1$) is called limiting case of BE degeneracy

Also the temp. at which value of $\frac{n}{V}$ is maximum is called critical temperature. i.e

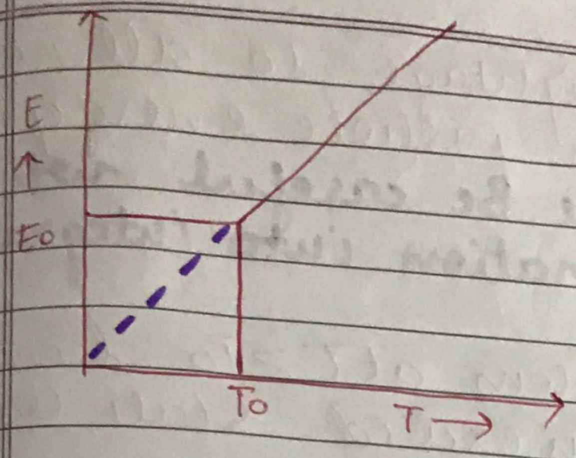
$$\boxed{T_0 = \frac{h^2}{2m\pi k}\left(\frac{n}{2.612V}\right)^{2/3}} \quad \text{(2)} \quad \text{from (1)}$$

For a value $\frac{n}{V}$ more than that given by (1) does not exist as ($A=1$ Max)

So there will be no solution of (1)

for $\boxed{T < T_0}$.

Hence T_0 is the critical temp. as the temp. at which degeneracy of energy level starts.



(i) Dotted line shows a non degenerate gas.

After To [at low temp] the number of particles begin to get crowded into the lower energy levels and a number of particles may occupy the ground state so we can not assume the energy distribution to be continuous as we have avoiding it and assuming it as such now.

Eqn- ② shows that η is a function of temp which is not correct. Something is wrong about our analysis since total number of particles should remain constant. So this eqn is not valid at $T < T_0$.

This arises :-

Because when we change summation into integration we miss the ground state i.e. $u=0$, we have ~~zero~~ given no weightage to ground state whereas quantum mechanically we should

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give equal weightage to all non degenerate single particle energy state. (Next time Be careful while replacing summation into integration)

It's not a problem at $T > T_0$ because the particles are in excited state but at $T < T_0$ particles tend to occupy ground state. And at absolute zero temp. all particles gathers to the ground state, and this phenomenon is called...

"BOSE EINSTEIN CONDENSATION".

So for temp $T < T_0$

$$N = N_0 + N_e.$$

(where N_0 is no. of particles in ground state
& N_e is no. of particles in excited state).

Now

$$N_e = V\left(\frac{2\pi m k T}{h^2}\right)^{3/2} \times 2.612 \quad \text{for } T < T_0$$

$$N = V_0\left(\frac{2\pi m k T_0}{h^2}\right)^{3/2} \times 2.612 \quad \text{for } T = T_0$$

because this equ is valid for $T = T_0$ and $T > T_0$

Dividing these two equation,

$$\frac{N_e}{N} = \left(\frac{T}{T_0}\right)^{3/2}.$$

$$N_e = N\left(\frac{T}{T_0}\right)^{3/2}.$$

so As, $N_0 = N - N_e$

$$\boxed{N_0 = N\left[1-\left(\frac{T}{T_0}\right)^{3/2}\right]}$$

This is the no. of particles in ground state.

As Temp ↑ N_0 ↓
Temp ↓ N_0 ↑

and At absolute $T=0$, all the particles are condensed into zero energy ground state. This is called Bose-Einstein Condensation.